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Congestion Pricing and Carmageddon



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by

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I hereby recognize and pledge to fulfill my responsibilities as defined in the Honor Code and to maintain the integrity of both myself and the College as a whole.

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ABSTRACT

The mathematical model for the Pigou-Knight-Downs paradox and the Downs-Thomson paradox, proposed by the economists Richard Arnott and Kenneth Small, explain why congestion fares in London have had a significant impact on reducing traffic in the city. However, their idea that solving these paradoxes through congestion pricing would lead to an indefinite cycle of increasing demand for public transportation is arguable. While this paper cannot conclude that there does not exist a significant correlation between congestion pricing and demand for public transportation, it will cast doubt on the accepted mathematical model described both in the Pigou-Knight-Downs paradox and the Downs-Thomson paradox. It lays a framework for proving why the mathematical model is wrong and the correct regression framework, which with the publication of more relevant data, can prove that the induced demand is moving somewhere else entirely.

I. Introduction

The popularization of mass production paired with steady decreases in oil prices over the last century has led to the cost of operating and owning a car dropping substantially. This development has led a large increase in the number of people who own cars in Western countries, such as Great Britain and the United States. While this may seem to be a benefit for mankind, many economists stress that this development should be feared.

Why should people have anxiety about this seemingly beneficial development? The main reason economists worry over this number is that traffic congestion has increased rapidly in major cities within the last fifty years. Traffic congestion is the increased use of space taken up on roads by vehicles. By decreasing the space for vehicles on roads, traffic congestion can cause lower average speeds for traffic and increase the amount of time it takes drivers to reach their destinations. Vehicular congestion in major cities can have a long list of unfavorable side-effects such as: enlarged costs to businesses, increased rates of traffic accidents, more air and noise pollution, and heightened rates of mental health problems for citizens (Arnott, 1994). All of these factors can have a substantial negative effect on economic growth in cities, so it is easy to see why city governments should be interested in lessening traffic congestion. However, how does one effectively accomplish reducing traffic congestion? Many governments have tried to combat congestion by increasing the capacity of roads by adding more lanes, but as will be shown in this paper, these efforts do not often work.

In February of 2003, the government of London enacted a road tolling policy called “congestion pricing” and saw an immediate reduction in the level of traffic in the

city (Leape, 2006). Their congestion pricing system set up automatic polls throughout Central London that forwards a cost to drivers for using certain roads. These costs vary depending on the hour of the day and reach a capping price range during peak-traffic hours (Leape, 2006). However, the effectiveness of this driving toll is still a topic of debate today. Many economists doubt how effective the congestion prices are in reducing vehicular traffic outside of Central London and argue that traffic may have just moved to the borders of the congestion pricing area (Leape, 2006). To combat views such as the one above, many supporters of the congestion pricing policy cite economists Richard Arnott and Kenneth Small's mathematical model as an explanation for how congestion pricing can effectively reduce traffic on main roadways and push more drivers to use public transportation.

This paper will dispute this argument proposed by supporters of the congestion pricing system in London. While there is statistical evidence that the policy did decrease traffic levels in and around Central London, there is no evidence that the policy caused drivers to turn to using the most integrated form of public transit in the city, the London Underground[1]. Specifically, this paper will target the main argument of Arnott and Small's mathematical models, that congestion pricing will cause an unlimited cycle of increasing demand for public transportation. Congestion pricing did have a substantial effect on reducing traffic on major roadways throughout London, but the expected increase in the use of the major public transportation system, the underground subway, has not been extensively researched by other economists and thus cannot be proven to be statistically significant (Leape, 2006). While this paper cannot conclude that there does not exist a significant correlation between London's congestion pricing policy and

[1] Public rapid transit system situated in London, England (Litman, 2004).

demand for London Underground, it will cast doubt on the accepted mathematical model described both in the Pigou-Knight-Downs paradox and the Downs-Thomson paradox. This paper will add onto existing research by laying a framework for proving why the mathematical model could be wrong as well as preparing the correct regression framework, which with the publication of more relevant data, can prove that the induced demand is moving somewhere else entirely

II. Literature Review

The first set of authors discussed in this paper are large contributors to the Pigou-Knight-Downs paradox and the Downs-Thomson paradox. These paradoxes are integral to both this paper and the general study of traffic economics. This includes the work of: Richard Arnott, Kenneth Small, M. J. H. Mogridge, Heti Afimeimounga, and Pengfei Guo. All of these authors talk lengthily about these paradoxes as well as the possible roles mispricing and induced demand have on causing traffic congestion. These authors contributed most to the theoretical section in this sense as their focus on these theories influenced my own. Richard Arnott and Kenneth Small's work is predominantly featured within this paper as their mathematical model for the two paradoxes are the main subject of this paper. While their analysis of congestion pricing has been influential to these authors as well as many other traffic economists, the doubt this paper places on its effectiveness will surely cause more scholastic debate among both its supporters and opponents. By questioning the effectiveness of these paradoxes in reproducing realistic outcomes, this paper challenges these authors to go over their own analyses and possibly create a more comprehensive and realistic mathematical model for these paradoxes.

The second set of authors used in this paper are influential in that their collective work helps provide a basis to explain congestion pricing as well as the other concepts behind congestion pricing's supposed usefulness. These authors include A. A. Walters, Todd Litman, Douglas Lee Jr., Richard Arnott, and the International Journal of Transport Economics. These authors wrote extensively during the early 1990s about concepts that effect congestion pricing and their combined work helped form this paper's conclusion that congestion pricing in London was most likely not efficient at increasing the number of passengers using London Underground. This paper adds onto the work of these authors by providing a mathematical interpretation of congestion pricing for Arnott and Small's paradox models. Arnott and Small have stated in their work that congestion pricing would be the best option to solve these paradoxes, but they never describe congestion pricing mathematically. This paper uses the work of Walters, Litman, Lee, and the International Journal of Transport Economics, to effectively describe congestion pricing within Arnott and Small's model, greatly adding onto its usefulness at describing scenarios. This development will hopefully push the mathematical models for the paradoxes to better predict what will happen in a realistic situation when congestion pricing is applied.

The last set of authors used in this paper give an empirical view on the mechanisms of the paradoxes and the traffic economic concepts previously discussed. The authors that contributed to this section of thought and data collection include: Andrew Evans, Gilles Duranton, Todd Litman, Jonathan Leape, Moshe Givoni, and Nathan Darroch. Their collective work explain a large amount about the effects of congestion pricing in major cities, London's congestion pricing system in 2003, and why

the London Underground subway system is one of the most important ulterior forms of transportation. These authors provided data on the effectiveness of the congestion pricing system in London on reducing traffic levels both in and around the area. This paper adds onto the work of these authors by taking their evidence and applying it to Arnott and Small's popular interpretation of the Pigou-Knight-Downs paradox and the Downs-Thomson paradox. The doubt this paper places on Arnott and Small's models brings up more questions for these authors as none of them have discussed where the missing traffic has ended up. By both posing this question and analyzing the possible options, this paper creates new opening for these authors to project their papers further and revisit London's congestion pricing system.

III. Theoretical Section

The economists Richard Arnott and Kenneth Small have greatly influenced many important concepts in traffic economics. One of their main contributions to the field may be their detailed analysis of the Pigou-Knight-Downs paradox and the Downs-Thomson paradox. In their analysis, Arnott and Small created a mathematical model that has been influential in explaining how the two paradoxes affect traffic congestion. However, while their work has been influential, doubt should be placed on their description of how congestion pricing can create a cycle of increasing demand for public transit systems such as the London Underground.

To better understand why doubt should be placed on both Arnott and Small's mathematical representations of the Pigou-Knight-Downs paradox and the Downs-Thomson paradox, induced demand, congestion pricing, and the paradoxes must first be introduced and explained in detail. The Pigou-Knight-Downs paradox will explain how a

congestion pricing policy can significantly decrease traffic, while the Downs-Thomson paradox will help clarify why some economists may have expected demand[2] for London Underground to have increased significantly.

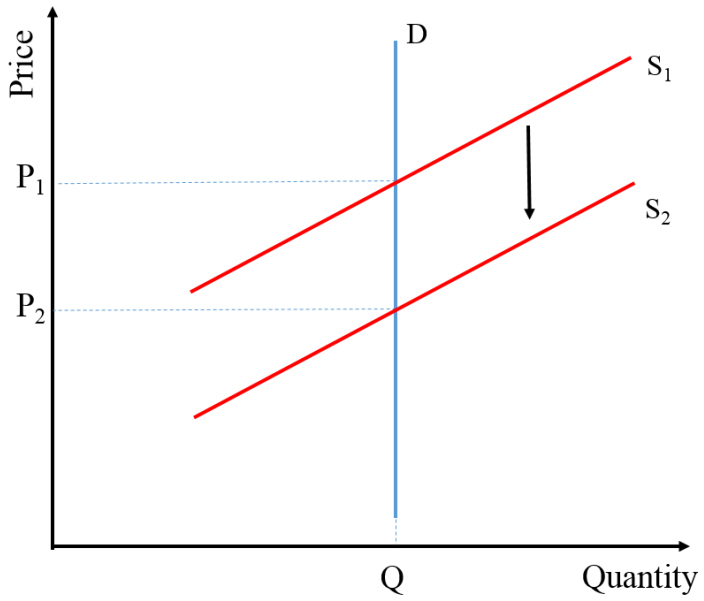
III.A Misconception about Traffic Demand

Before the introduction of properties like induced demand, many economists treated the supply and demand of drivers in a misconstrued manner, such as in *Figure 1*. This figure illustrates the common misconception that road capacity increases will not cause any change in the quantity demanded of that road. The model defines the relationship between the cost of traveling on a specific road, P[3], and the quantity consumed, Q[4]. The main difference from this graph to one that properly explains the induced demand of traffic is the elasticity of the demand and supply curve (Evans, 1992). The demand curve, represented by *D*, is totally inelastic and the supply curve, denoted as *S*, is elastic at a constant rate. The demand curve is characterized as being inelastic to represent the idea that a shift in price will not change the quantity demanded of the road. The supply curve is represented as being isoelastic[5] to illustrate the misconception that a change in the price of using the road will always cause the same change in the quantity supplied of the good. With this misunderstanding about the elasticities of the demand and supply curve, economists concluded that if supply increased, such as from S_1 to S_2, the time cost it takes to use that road also decreases (Evans, 1992). However, this explanation does not fit the conclusions often found in reality, for when the total supply of a road

[2] In this situation we use the term demand instead of quantity demanded because the total demand for a service is changing
[3] Measures the time it takes to get to the destination.
[4] Measures the change in the number of vehicles using that specific road.
[5] Constant elasticity function.

increases, at many times there seems to be an increase in the quantity demanded as well. Thus, this viewpoint on the elasticity of the supply and demand curve is inefficient as it does not take in to account the complex behavioral incentives of drivers.

Figure 1: Misconception about Traffic Demand

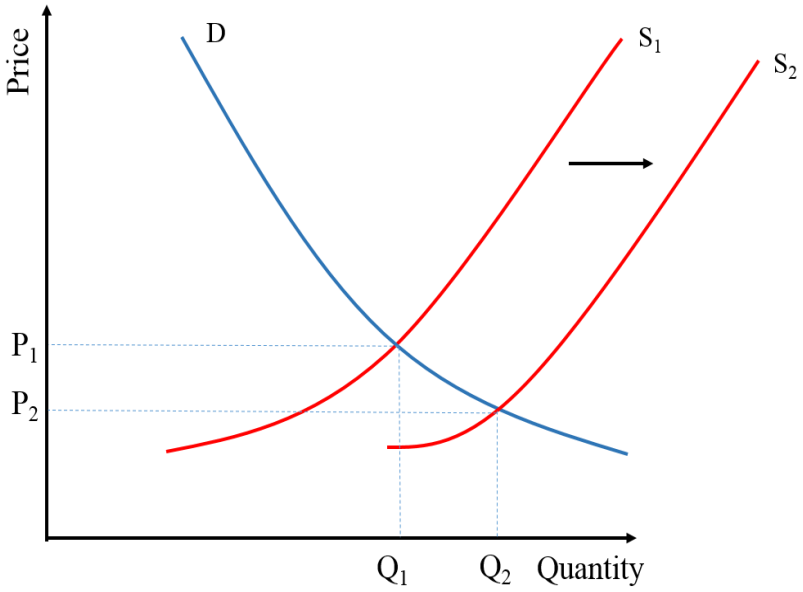


III.B Induced Demand

To better define what was happening with road capacity increases, economists began turning to the property of induced demand. This term refers to the manifestation of increased quantity of consumption for a good or service when the supply of that good or service increases. Traffic economists began using the term widely in the 1950s, during the forefront of highway development in the United States (Lee, 1999). The economists found that when extra lanes were added to these highway networks, there often was an increase of the quantity of cars using the highways. Hence, economists began treating the

demand for space on congested roads as being induced (Litman, 2014). Unlike in *Figure 1*, the idea of induced demand curve takes these complex behavioral patterns of drivers into consideration and concludes that while a total supply increase can reduce the price of using the road, the quantity demanded will also increase (Evans, 1992). Thus, the increase in quantity demanded will reduce the effectiveness of the total supply increase in reducing the time cost it takes to use the road efficiently.

Figure 2: Induced Demand



The significance of the elasticity of the demand and supply curve is illustrated in *Figure 2*. When the capacity of a road increases, represented by the shift from S_1 to S_2, the cost[6] of using the road also decreases, illustrated by the movement of P_1 to P_2. As the cost decreases, the quantity of drivers on the road increases, which is represented by the movement from Q_1 to Q_2. Thus, the initial increase in road capacity brings about an increase in the quantity of demand for that road. The increase in the quantity demanded reduces the effectiveness of the increase in total supply for decreasing the time cost (Evans, 1992). Thus, both the demand and supply curve should be curved to represent the varying elasticity rate that gives way to this conclusion.

III.C Pigou-Knight-Downs Paradox

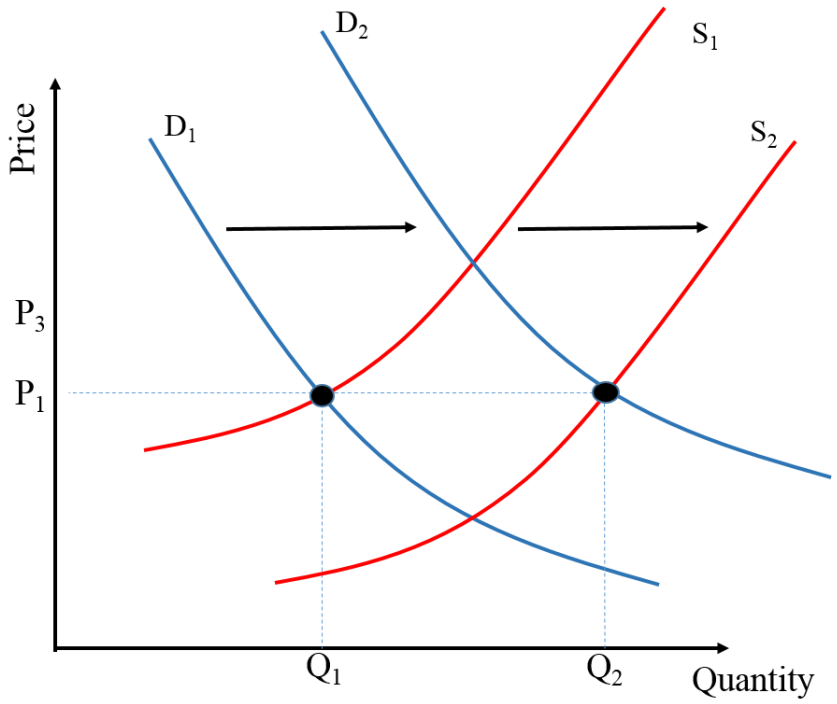
It is important to describe the Pigou-Knight-Downs paradox before the Downs-Thomson paradox as it helps explain how the latter's economic setting comes to fruition. The main reason why the Pigou-Knight-Downs paradox is important is that it builds upon the idea of induced demand and helps explain why the government of London would be inclined to use congestion pricing instead of increasing road capacity in an effort to reduce traffic levels. The Pigou-Knight-Downs paradox places a large focus on examining the behavior of common drivers and how they may change their preferred driving routes based on increases or decreases to traffic levels[7].

[6] In this case, the cost is seen as the amount of time it takes for a driver to get from their starting point on the road to their ending destination on the same road

[7] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

The Pigou-Knight-Downs paradox explains why an increase to the capacity of a road can commonly have no effect on the traffic levels of that specific street. Accordingly, the main factor that separates the doctrine of the Pigou-Knight-Downs paradox from induced demand is that not only will increasing road capacity lead to the quantity of space demanded on that road to increase, but it is possible that the increase will not lead to any reduction in the cost it takes to use the road[8].

Figure 3: Pigou-Knight-Downs paradox



[8] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

The Pigou-Knight-Downs paradox is in many ways an expansion of the idea of induced demand. The paradox parallels the notion of induced demand for busy roads, as it too states that increasing road capacity will in some cases lead to an increased level of quantity demanded for that road. Thus, in a graph that represents the Pigou-Knight-Downs paradox, such as *Figure 3*, looks similar to a graph that denotes induced demand, much like *Figure 2*. The starting point for *Figure 3* is the same as *Figure 2*, represented where P_1 meets Q_1. Again we address the scenario in which the supply of space on the road increases due to the addition extra lanes. As with induced demand, the cost for drivers to use the road will decrease and the quantity of space demanded on the road will increase. However, the Pigou-Knight-Downs paradox[9] argues that induced demand will cause total demand to increase, as is illustrated by the demand curve, denoted as D_1, shifting to D_2. This is where the paradox distinguishes itself from the general theory surrounding induced demand.

With the increase in total demand, the size of the initial supply increase is paramount in determining whether the cost will be move back to P_1. If the initial supply increase is small enough, the paradox claims that the price will move back to point P_1 the original cost before the road capacity increase. Thus, the Pigou-Knight-Downs paradox does parallel the ideas of induced demand by stating that dormant drivers will reduce the effectiveness of road capacity increases by migrating to the road space as it becomes available. However, the paradox differentiates itself by arguing that through induced demand, the cost for drivers could become the same as it was before the initial increase. The paradox is a strong argument for avoiding road capacity increases if one is trying to

[9] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

reduce traffic congestion, and supporters of the paradox throw backing behind the idea of congestion pricing on reducing traffic.

III.D Pigou-Knight-Downs Paradox Mathematical Model

To explain the Pigou-Knight-Downs paradox mathematical model[10], assume that there are *D* number of drivers that all desire to get from point *A* to point *B* (Arnott and Small, 1994). These *D* number of drivers have two routes to choose from to get from point *A* to point *B*. Let these routes be known as route *X* and route *Y*. Now assume that route *X* is a narrow bridge that takes *M* amount of time to cross if traffic is free-flowing[11]. Route *Y* is a wide highway that takes twice amount of time to cross no matter the traffic situation on the route (Arnott and Small, 1994). In any case like this, without any other externalities, it is certain that the drivers will always choose route *X* over *Y* as *X* is the route that gets the drivers to their desired destination the fastest. One can take this to be true by assuming that the drivers act like rational consumers. The term "rational consumer" refers to the rational choice theory, which will be assumed to be true in this scenario. This theory states that consumers will always choose the most rational option, that is, the option that maximizes their satisfaction or well-being. By assuming this theory, it makes sense that the drivers would prefer to choose route *X* over route *Y* when there are no other costs besides time, as drivers will desire the option that minimizes their costs.

The mathematical model only needs to compare these two routes to reach a realistic conclusion about route options as route *Y* symbolizes the sum of all the other

[10] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.
[11] The optimal speed of the road without any traffic congestion.

travel options that the drivers have to reach point *B* (Arnott and Small, 1994). This summation of traffic options simplifies the equation for driving options and still keeps the equation comparable to real world examples. Thus route *X* represents the one route that has the overall quickest travel time, while route *Y* is the summation of all the other options for the drivers that take longer than route *X*.

In this mathematical model, the capacity of the two routes is defined as the number of drivers in which the speed drops to half of what it would originally be without any traffic. Route *X* has the characteristic that travel time rises linearly in relation to the number of drivers divided by total capacity. This relation only applies to route *X* as route *Y* has infinite capacity so any change in the number of drivers using it will not change the amount of time it takes to get from point *A* to point *B*. Since route *X* represents the one most efficient route to get from point *A* to point *B*, it must have a relation to measure the effect of increasing congestion.

For the two routes, T_1 will represent total travel time for route *X* to travel from *A* to *B* while T_2 will represent the same relation for route *Y*. Let F_X be the traffic flow using route *X* and F_Y be the traffic flow using route *Y* such that the sum of F_X and F_Y equals the total number of drivers *D*. Finally, C_X will represent the capacity of route *X* such that F_X divided by C_X equals the ratio of traffic flow using route *X* to total capacity on route *X*[12].

$$T_1 = M + M\left(\frac{F_X}{C_X}\right)$$

$$T_2 = 2M$$

[12] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

$$F_X + F_Y = D$$

Reviewing the mathematical model, one can see that the travel time for each route is equal when the capacity of route *X* is equal to the flow of traffic on route *X*. In this case, rush hour drivers can choose either option as they both deliver the drivers to their destination in the same amount of time. Reviewing this fact, there are two important cases that are significant in illustrating how capacity increases affect traffic congestion. The two important examples are observable when the capacity for route *X* is either greater than *D* or less than *D*[13].

$$F_X = C_X$$

$$T_1 = M + M\left(\frac{F_X}{C_X}\right) = 2M$$

$$T_2 = 2M$$

$$T_2 = 2M = T_1$$

When the capacity for route *X* is greater than *D* we see that the paradox does not exist and any increases to capacity will decrease the amount of time that it takes for rush hour drivers to reach point *B*. This is true as the road capacity is now much larger than any value that F_X could possibly be so T_1 cannot possibly be equal or lower than T_2. It is also true that the amount of travel time it takes to cross route *X* decreases at a reducing

[13] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

rate, such that the first few additions to capacity still has a large marginal effect on reducing traffic congestion.

$$F_X < C_X$$

$$T_1 = M + M\left(\frac{F_X}{C_X}\right) < 2M$$

$$T_2 = 2M$$

$$T_2 > T_1$$

The Pigou-Knight-Downs paradox becomes relevant when the capacity of the route is value below the number of total drivers. At this point, any additions to capacity that does not increase total route capacity greater than *D* is not effective at reducing traffic[14]. If the capacity of route *X* is increased to any value less than *D*, drivers that used to use route *Y* will move to route *X* as it is the most efficient solution to get from point *A* to point *B*. This split is a natural occurrence for traffic as per the rational consumer theory, as drivers will seek to minimize their time cost and thus will continue to switch over to route *Y* until the total amount of time it takes to use route *X* reaches 2*M*. Thus, the only radical change from the capacity increase would be that the ratio of drivers using route *X* to route *Y* has increased.

[14] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

$$F_X > C_X$$

$$T_1 = M + M\left(\frac{F_X}{C_X}\right) > 2M$$

$$T_2 = 2M$$

$$T_2 < T_1$$

III.E Congestion Pricing

Arnott and Small's mathematical model argues that congestion pricing is the most efficient way to decrease traffic on major roadways. Congestion pricing is a relatively new tolling system that has been implemented in a number of metropolitan areas in the world such as Singapore and Stockholm, Sweden. It is similar to a bridge or road toll in that it taxes vehicles for using certain stretches of roadway. However, congestion pricing differs from other tolls in that the value of its toll depends on the time of day, with the highest toll value paralleling the time of day that sees the most traffic on the roadway. Thus, the highest values of the toll usually are centered on times when commuters usually go to and from their place of work (Evans, 1992).

The reason why the Pigou-Knight-Downs paradox supports congestion pricing is that it solves the mispricing of traffic. Economists believe this mispricing naturally exists for traffic as drivers don't usually have a cost associated with adding to an already congested road. Thus, drivers will continue to use a road and add to the total time cost for as long as that specific road is the most efficient at getting them to their destination. By

placing a cost on each additional unit of traffic congestion pricing fixes this problem and allows drivers who have a higher willingness-to-pay to use the road over those that have a lower willingness-to-pay. The term "willingness-to-pay" refers to the maximum price that one would pay for a good or to forgo something unwanted (Walters, 1961). Without congestion pricing, drivers with a lower willingness-to-pay for road space would use the same roads that other drivers with higher willingness-to-pay also demanded. Consequently, those that would pay more to use the road and possibly get to their destination quicker were instead hindered by those who place a lower value on using the road. Thus, by implementing congestion pricing, drivers with lower willingness-to-pay values are instead incentivized to use other roads (Walters, 1961).

$$T_1 = M + M\left(\frac{F_X}{C_X}\right)$$

$$T_2 = 2M$$

$$TC_2 = 2M$$

$$TC_1 = M + M\left(\frac{F_X}{C_X}\right) + CT(V)$$

III.F Downs-Thomson Paradox Mathematical Model

The Downs-Thomson paradox is where the argument of this paper bases its criticisms. The paradox is similar to the Pigou-Knight-Downs paradox in that it focuses

on congestion and the ill effects of adding additional capacity to roads as well as stating that congestion pricing can effectively solve the induced demand and mispricing found on busy roads. However, in the case of the Down-Thompson paradox, it compares private transportation with public transit and argues that congestion pricing will lead to an indefinite cycle of increasing demand for public transit use (Mogridge, 1987).

For this paradox, assume that the public transit in question is a subway system, such as London Underground, while the private transportation is route *X* from the previous example that represents a narrow bridge for vehicular traffic. It is also assumed, like in the previous example, that the total number of drivers is equal to *D* and the drivers are trying to get from point *A* to point *B*[15]. Since the train is a part of public transit, it follows that its profits increase when the volume of users increases. The main cost to most forms of public transportation is operation of the transit vehicle, in this case, the electricity needed to run the generators to make the train move. Thus, it follows that for a public train that has numerous cars to hold passengers, the most cost effective way to construct route pick-up times is based on whatever waiting time allows them to fill their cars to their maximum holding value (Guo, 2014). By filling their cars to their maximum, they can increase ticket revenue per a train ride, reducing the cost of operating a train by comparison.

For this example, it is assumed that the train takes 2*M* to get people from *A* to *B* while the same equation for route *X* in the Pigou-Knight-Downs paradox will be reused to define route *X*. Thus, it is presumed that route X once again has the quality that travel time rises linearly in relation to the number of drivers divided by total capacity (Arnott

[15] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

and Small, 1994). Now suppose that for every three times *D* passengers, the travel time for the public transportation option will decrease a total of *M*. One can assume this as public transit is the most cost efficient when it fills its cars before leaving. Thus, the more passengers there are waiting for the train, the less time the train has to wait to fill its cars[16].

The time it takes to use the train is represented by T_T and the flow of traffic for the train is F_T. The equation for the amount of time it takes the train to go from point *A* to point *B* is the flow of traffic divided by $3D$[17] and multiplied by *M* (Arnott and Small, 1994). This is then all subtracted from $2M$ to get the total amount of time it takes for the train to transport its customers.

$$T_X = M + M\left(\frac{F_X}{C_X}\right)$$

$$T_T = 2M - M\left(\frac{F_T}{3D}\right)$$

$$F_X + F_T = D$$

$$F_X = D - F_T$$

There are two important examples of this paradox; the first has to do with when the capacity of the road is less than *D* and the second is when the capacity is greater than *D*. When the road capacity is less than *D*, the mathematical model shows that the

[16] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

[17] The value 3D (D multiplied by 3) used in this example is of random choice by the author. The value must be a greater value than D to illustrate that the number of passengers it would take for there to be a substantial decrease in travel time for the train would be quite large. This is supported by the work of Arnott, R., & Small, K. (1994).

commuters will spread themselves out over the differing options until the time it takes to take private transportation equals that of public transportation. With this assumption, we set both T_X and T_T equal to each other. The model states this as true as the spread of traffic will reach an equilibrium when the capacity of the road is too low[18].

$$T_X = M + M\left(\frac{F_X}{C_X}\right) = 2M - M\left(\frac{F_T}{3D}\right)$$

$$T_X = M + M\left(\frac{F_X}{C_X}\right) = 2M - M\left(\frac{(D - F_X)}{3D}\right)$$

$$\left(\frac{DC_X - F_X C_X + 3DF_X}{3DC_X}\right) = 1$$

$$F_X = \frac{2DC_X}{3D - C_X}$$

One can simplify Arnott and Smalls model by setting the two equations equal to each other and pushing the value of capacity to one side and the flow of traffic on route *X* to the other side. In the end, an equation like that shown above is created. The Downs-Thomson paradox occurs when the increase to road capacity for route *X* does not push the total road capacity past *D* number of drivers. For this example let us assume that the road capacity of route *X* is equal to half of *D* number of drivers. The equation shows that when capacity equals this, free flow traffic on route *X* will be equal to two-fifths of *D*. There is now enough information to calculate the amount of time it would take to cross the bridge, which comes out to be nine-fifths *M*. To find the time it takes to use public transit, one

[18] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

must first find the free flow rate of traffic for the train. Since the number of drivers is set at *D*, the free flow of traffic for the train is found by subtracting the free flow on route *X* from *D*, which comes out to be three-fifths *D* drivers. By plugging in the free flow value for the train, one can find that the total time it takes to use the train is also nine-fifths *M*.

$$F_X = \frac{2D(\frac{1}{2}D)}{3D - (\frac{1}{2}D)}$$

$$F_X = \frac{2}{5}D$$

$$F_T = D - \left(\frac{2}{5}D\right) = \frac{3}{5}D$$

$$T_X = M + M\left(\frac{(\frac{2}{5}D)}{(\frac{1}{2}D)}\right) = \frac{9}{5}M$$

$$T_T = 2M - M\left(\frac{\left(\frac{3}{5}D\right)}{3D}\right) = \frac{9}{5}M$$

Now suppose that the there is an increase of capacity that still keeps total capacity for route *X* below *D*. Suppose that to reduce the time it takes for commuters to reach point *B*, the local government decides to increase the capacity of route. Thus the government proceeds by increasing the number of traffic lanes on route *X*. Will this have a significant impact on decreasing traffic? No, as the capacity does not increase to over

D. If the capacity instead rose from one-half of *D* to three-fourths of *D*, the time it takes for both public transit and route *X* to get to point *B* increases to seventeen-ninths *M*[19].

$$F_X = \frac{2D(\frac{3}{4}D)}{3D - (\frac{3}{4}D)}$$

$$F_X = \frac{2}{3}D$$

$$F_T = D - \left(\frac{2}{3}D\right) = \frac{1}{3}D$$

$$T_X = M + M\left(\frac{(\frac{2}{3}D)}{(\frac{3}{4}D)}\right) = \frac{17}{9}M$$

$$T_T = 2M - M\left(\frac{\left(\frac{1}{3}D\right)}{3D}\right) = \frac{17}{9}M$$

The increase in capacity pulls the induced demand from the train over to route *X*. So the free flow of traffic for route *X* increases from two-fifths *D* to two-thirds *D* drivers while the free flow of traffic for the train decreases from three-fifths *D* to one-third *D* drivers. This decrease in the number of people using the train also decreases the efficiency of public transit, as the operator of the train wants to fill the train cars to capacity before leaving to increase revenue per a trip. This desire to increase revenues per

[19] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

a trip, will slow down the service of the train as is shown in the equation above. This phenomenon will continue as road capacity increases, with induced demand for route *X* pushing more and more commuters to take the additional capacity created on route *X*. Thus the travel time for each option increases until the capacity of route *X* equals or is over *D* commuters. At this point, all of the drivers will switch over to route *X* as the capacity makes the time it takes to get to point *B* equal for both routes.

When the capacity increases over *D* commuters, then the paradox no longer exists and the increasing capacity leads to a decrease in the time it takes for rush-hour commuters to reach point *B*. If the capacity increases to two times *D* then the flow of traffic over route *X* will still be equal to *D* drivers, as this is the maximum of induced and available demand. In this case, the time it takes to cross will equal three-halves *D* and the free flow of traffic for the train will remain at zero. Since the flow of traffic for the train is zero commuters, the total time it takes for the train to get from point *A* to point *B* will be two times *M*[20].

$$F_X = \frac{2D(2D)}{3D - (2D)} = < D$$

$$T_X = M + M\left(\frac{D}{2D}\right) = \frac{3}{2}M$$

$$T_T = 2M - M\left(\frac{0}{3D}\right) = 2M$$

The Downs-Thomson paradox explains how it is possible for an increase in capacity of a roadway to also increase the congestion levels for that roadway. This

[20] Arnott, R., & Small, K. (1994). The Economics of Traffic Congestion. *American Scientist, 82*(5), 446-455.

congestion can increase the amount of time it takes to use the roadway to get to a destination. One may argue that it is beneficial to increase the capacity when it crosses the equilibrium point such as when capacity is over *D* drivers in the Pigou-Knight-Downs paradox and the Downs-Thomson paradox. However, as is seen within both paradoxes, the demand for the roadway comes from individuals who don't use the good until capacity is raised to a certain level. Therefore, it is extremely difficult to measure the level of capacity that will satisfy induced demand as there is no way of telling what level of latent demand exists for the roadway (Afimeimounga, 2005). This paradox proves how risky it is to try to decrease traffic congestion by increasing roadway capacity. As has been proven by these paradoxes, capacity increases are do not always have a positive effect and can be prone to backfire by increasing traffic congestion.

The Downs-Thomson paradox, just as with the Pigou-Knight-Downs paradox, argues that that congestion pricing is the most efficient way of solving the mispricing of traffic and pushing drivers with a lower willingness-to-pay to use the ulterior route. However, in this case, Arnott and Small's mathematical model for the Downs-Thomson paradox introduced the idea that the efficiency of the public transit system would increase as more drivers were pushed to use it. By solving the mispricing of traffic and pushing the people with a lower willingness-to-pay for road space to take public transit, gradually the effectiveness of the public system would increase and the total cost would fall. This fall in total cost should push more drivers that have a low willingness-to-pay for space on route X, to take the public transit system, in this case, London Underground. This should create an indefinite cycle of increasing public transit efficiency and demand for space on the public transit system.

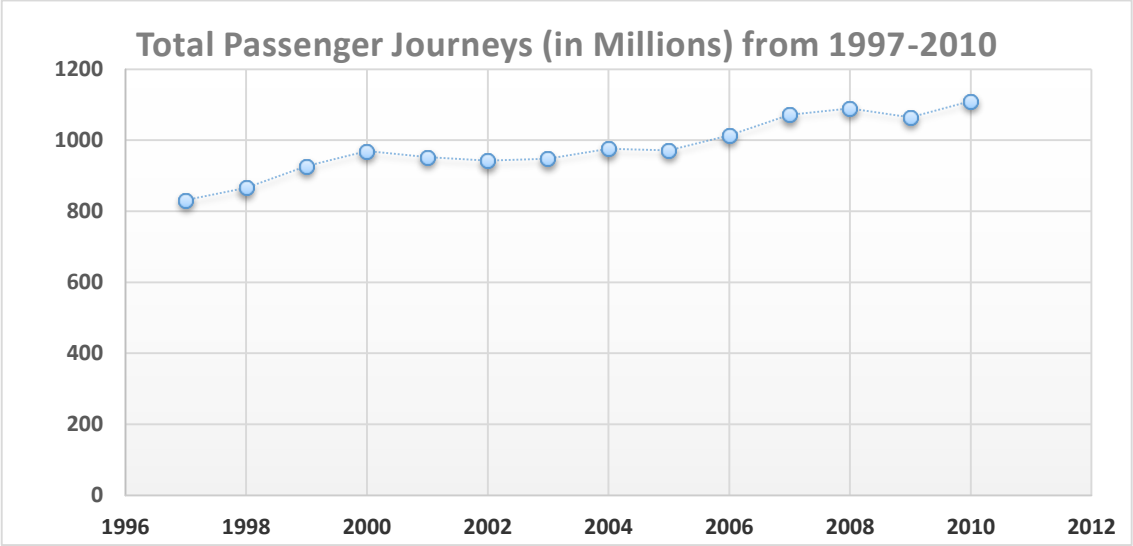
$$T_T = 2M - M\left(\frac{F_T}{3D}\right)$$

IV. Empirical Section

The congestion pricing system in London was at reducing traffic levels within the congestion pricing area. By implementing a congestion fare, the mispricing on the congestion in London has improved and thus the average speed of vehicles and travel time within the congestion pricing area also improved (Afimeimonga, 2005). Thus the Pigou-Knight-Downs paradox's and the Downs-Thomson paradox's argument that congestion pricing is the best option for decreasing traffic levels does carry some merit. However, the claim made by Arnott and Small's models that solving the mispricing of traffic would cause a loop of increasing train efficiency remains to be seen (Afimeimonga, 2005).

This paper's mistrust of their conclusion on public transit is not misplaced. As can be seen in Figure 5, there doesn't seem to be any large increase in London Underground ridership before or after congestion pricing was enacted in London.

Table 1: Total Passenger Journeys



Source: Transport for London, London Underground

While there is a slow rise in the number of passengers using London Underground, there is no significant spike in ridership. This is important as the indefinite cycle of rising train efficiency and demand for the train that is defined in the Downs-Thomson paradox should cause a substantial and visible increase. The fact that this spike in ridership does not exist, places doubt on how well the Arnott and Small's models are at describing realistic situations.

One may cite the Pigou-Knight-Downs paradox and state that the missing demand may have just moved to the next most efficient road instead of switching over to using public transit. However, Leape finds in his regression on the effectiveness of congestion pricing in reducing traffic, that traffic rates did not rise on main roads surrounding the congestion pricing area (Leape, 2006). This is important as the next most efficient route for these drivers is most likely these roads. Mathematically, if going through the

congestion area was most efficient for these drivers, the second most efficient driving option must be in an area very close to it (Walters, 1961). The traffic on these routes did not change significantly however, so the drivers with a lower willingness-to-pay must have used some ulterior form of transportation.

It makes sense that the London Underground would be the most popular form of ulterior transportation as the subway system is already rated as one of the most popular ways to travel through London and its network is spread throughout all of inner and outer London (Leape, 2006). Thus, London Underground seems to be the most viable option for the induced demand if the drivers did not move to their second most efficient car routes.

IV.A Regression Analysis

In an effort to prove that Arnott and Small's mathematical model did not effectively predict the demand level for London Underground, this paper produced a regression to see if congestion pricing had any substantial effect on increasing use ridership for the subway system. The findings of the economists and Leape and Litman do support the notion put forward by the paradoxes that congestion pricing would be have a substantial effect on reducing traffic. This paper's argument is that the mathematical models do not effectively predict where these missing drivers will end up.

Table 2: Regression and Regression Equation

Source	SS	df	MS	Number of obs =	11
				F(4, 6) =	22.14
Model	3.6737e+16	4	9.1842e+15	Prob > F =	0.0010
Residual	2.4884e+15	6	4.1474e+14	R-squared =	0.9366
				Adj R-squared =	0.8943
Total	3.9225e+16	10	3.9225e+15	Root MSE =	2.0e+07

TotalJourn~s	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
CongYears	-3.16e+07	3.21e+07	-0.99	0.362	-1.10e+08	4.69e+07
GovSpending	7823924	1.69e+07	0.46	0.660	-3.36e+07	4.93e+07
PopLondon	109.3878	139.5339	0.78	0.463	-232.0394	450.8151
Fare	6.67e+07	2.68e+07	2.49	0.047	1123504	1.32e+08
_cons	-3.79e+07	9.55e+08	-0.04	0.970	-2.38e+09	2.30e+09

$$TotalJourns = \beta 0 + \beta 1(CongYears) + \beta 2(GovSpending) + \beta 3(PopLondon) + \beta 4(Fare) + \varepsilon$$

The regression used in this paper uses the annual number of single fare passenger journeys on London's subway system as the dependent variable, represented as *TotalJourns*. One of the independent variables used in this regression is a dummy variable that is equal to zero for the years before the application of the congestion pricing. This variable, denoted as *CongYears*, is included in this regression because if there exists statistically significant relationship between *CongYears* and *TotalJourns*, it would help explain if the induced demand for the use of roads in Central London moved to London Underground. The second independent variable included in this regression is the amount of government spending London funneled into public transportation per year, symbolized as *GovSpending*. This variable is included in the regression in an effort to prove that increases in public spending did not have any significant impacts on passenger levels for

London Underground. The next independent variable included in the regression is the annual census for the population of London, denoted as *PopLondon*. This variable is included in the regression in an effort to make sure that any significant increases or decreases in passengers did not stem from the number of people moving in and out of London. Finally, the last variable included in the regression is the annual fare for a single trip for London Underground, represented as *Fare*. The inclusion of this independent variable in the regression is important as it helps illustrate that changing fare prices did not have too much of a significant impact on changing London Underground use. These independent variables are included in this regression as they effectively make up many of the plausible variables that may affect the number of people using London Underground. The

In regressions, the R-square value expresses the variance of the dependent variable *TotalJourneys* explained by the independent variables included in the regression. For this regression, the R-square value is 0.9366, which states that this regression model explains 93.66% of the variance of London Underground passenger journeys. However, this estimate can be biased or over fitted with too many terms. One can look to the adjusted R-squared to see if the regression model is biased, for if the value is not high or close to the R-squared value, then there is a high chance that there are some independent variables that are not needed in the model. In this specific regression the adjusted R-squared value is 0.8943, which is both high and close to the 0.9366 R-squared value so one can conclude that the variables used in this regression model are efficient.

Table 3: Regression Statistics

Regression Statistics	
Observations	11
R-Squared	0.9366
Adjusted R-Squared	0.8943
F(4, 6)	22.14
Prob > F	0.001

However, through further examination, the regression model turns out to be not noteworthy in producing any significant data for these variables. This is illustrated by the F-ratio for the regression model and the p-values for each independent variable. The F-ratio tests whether the regression is a good fit for the figures provided. Since *F(4, 6) = 22.14* and the *Prob > F =0.0010* (which is higher than the chosen alpha $\alpha=0.05$). Thus, the output shows that the number of people using the London Underground subway system is not statistically predicted by the data for the independent variables given.

The p-values are important for this regression as they allow us to state that there is strong evidence for rejecting the null hypothesis. The null hypothesis is the proposal that the observations do not produce any relevant data are instead attained purely by chance. The null hypothesis can be rejected if the p-value is less than the chosen alpha of 0.05. If it's greater than this value, then the null hypothesis cannot be rejected and there remains a large chance that the findings of the regression are a result of chance and are not significant.

The regression's p-values for the independent variables of *CongYears*, *GovSpending*, *PopLondon* are all significantly greater than 0.05, so their coefficients are found to be not statistically significant are influenced more by random chance than by

their relationship to total passenger journeys for London Underground. However, the p-value for *Fare* is surprisingly significant. It is close to 0.05, as it is at a 0.047 p-value, but it is still viewed as an insignificant development as it does not make sense that an increasing fare would cause the number of London Underground passengers to increase.

Table 4: Independent Variables from Regression

Independent Variables	*Coefficient*	*P-value*	*Reject Null Hypothesis?*
CongYears	-3.16E+07	0.362	No
GovSpending	7823924	0.66	No
PopLondon	109.3878	0.463	No
Fare	6.67E+07	0.047	Yes

IV.B Regression Prediction

While the regression model makes sense, the main reason why this regression model is not significant for making conclusions is that the number of observations are very low. The data collected for this regression is based on annual collections and can only be place over the years of 1997 to 2011, which does not give enough information for making any significant conclusions. Thus, if there exists data for the months or days from the beginning of 1997 to the end of 2011 for all the variables, a regression could be created that could produce a significant conclusion on whether Arnott and Small's models are wrong.

Table 5: Prediction of Independent Variables

Independent Variables	*Expected Coefficient Sign*	*Coefficient Size*
CongYears	Positive	Very Small
GovSpending	Positive	Small
PopLondon	Positive	Small
Fare	Negative	Large

If more data was available for the variables *TotalJourneys*, *GovSpending*, *PopLondon*, and *Fare*, this paper theorizes that the model would become significant and a conclusion could be reached. Following the theories and paradoxes introduced in this paper, one could predict whether the coefficients for each independent variable would be negative or positive and how large they may be.

This paper predicts that the expected coefficient on *CongYears* is positive yet underwhelming small, as the Downs-Thomson paradox argues that the drivers that do not want to pay the congestion pricing toll in Central London would arguably move to ulterior modes of transportation. While on would predict that some drivers would be persuaded to use London Underground because of the congestion pricing system, this paper does not believe that Arnott and Small's theory of a cycle of increased efficiency and demand for the subway appeared. Thus, this paper argues that while the coefficient should be positive, it should be small and insignificant for increasing use of London Underground.

The coefficient for the independent variable of *GovSpending* is expected to be a small positive number as it makes sense that investment into public transit would have at a minimum a small positive effect on ridership of the London Underground. Drivers would much more likely be persuaded to use London Underground than stop their use of

it if the city of London increases their investment in it. However, one would expect that the coefficient would be very small and insignificant for influencing ridership.

The independent variable *PopLondon*'s expected coefficient would most likely be a small positive number. The expected coefficient sign is positive because of the idea that as more people move to London, one would expect that the people would spread themselves across all forms of transportation. Thus, there would be some form of positive correlation between an increasing population in London and an increasing rate of people using London Underground. However, this number would most likely be small and insignificant for increasing the amount of riders using London Underground.

Finally, the expected correlation for *Fare* is negative as one expects that as the price for using London Underground increases, the demand for using the London Underground will decrease. The coefficient for this variable should be relatively large as one suspects that a minor increase in price would cause the cost of using London Underground to go above a large amount of people's willingness-to-pay. One also suspects that this variable will be significant in reducing and increasing the number of London Underground passengers because of the reason provided above.

V. Conclusion

While this paper's regression did not pinpoint any significance conclusion and thus cannot disprove Arnott and Small's mathematical models for the Pigou-Knight-Downs paradox and the Downs-Thomson paradox, this paper does provide a framework for the correct regression and the theoretical structure that could help disprove their mathematical model if more data becomes available in the future. The Pigou-Knight-

Downs paradox and the Downs-Thomson paradox are effective in explaining why the congestion pricing system in London reduced vehicular traffic, but since influential traffic economists such as Leape and Litman also doubt that the congestion pricing system had any substantial effect on London Underground ridership, there is little support for their mathematical model (Litman, 2004). The framework for Arnott and Small's mathematical model states that there should exist a large cycle of increased demand for the London Underground after the congestion pricing was put in place. However, just from viewing London Underground ridership year-by-year, there doesn't seem to be any visual indication that this strong, indefinite cycle ever appeared. While ridership does rise from the year congestion pricing was introduced onwards, the fact that this cycle is not visible provides some doubt of its existence.

If the proper data becomes available, Arnott and Small's mathematical model will mostly likely be inefficient for fully describing realistic traffic situations and thus will need to be revised. The fact that many economists already use their mathematical models as arguments to enact congestion pricing policies and withdraw government spending on public transit is concerning. If the required data becomes available and the predicted public transit does not appear, congestion pricing supporters will need to conduct more studies. These supporters must find where these missing drivers are moving too, as this will remain as a significant mystery until more study is done on the effects of the congestion pricing system in London.

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