

The Evolution of Partially Disrupted Supergiant Stellar Remnants

Hydrodynamic Simulations of Star-Black Hole Collisions



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Abstract

Tidal Disruption Events (TDE's) are the violent gravitational interactions between stars and black holes, which usually occur in dense star clusters and galactic nuclei. The focus of this research is of Smoothed Particle Hydrodynamic simulations of Partial Tidal Disruption Events (PTDE's) where a $24 M_{\odot}$ blue supergiant, made in MESA, passes sufficiently close to the black hole as to not get destroyed completely, rather the inner layers or core of the star could remain intact after a considerable amount of mass is stripped from the surviving remnant. This remnant, along with any stripped mass, can either be still gravitationally bound in an elliptical orbit around the black hole post-interaction, or have enough kinetic energy imparted into it so that it becomes ejected from the system altogether. This project finds that PTDE's with a black hole of similar mass will lead to tidal capturing and eventual inspiral. In addition, while PTDE's with a higher black hole mass to star mass ratio have a higher probability of kicking the remnant out of the system with a high velocity, I've found that this result (which is generally true for main sequence stars) is not entirely true for supergiant stars. Another focus of this research is the evolution of the stellar remnant along with its composition, which I have found does not differ significantly from a star of the same mass and radius when considered at the surface.

1 Introduction

1.1 Motivation

The focus of this research is gravitational interactions of a blue supergiant star with a black hole of varying mass. Interactions of this kind are exceptionally rare and hard to spot, although we do have observational and computational evidence of their occurrence within dense star clusters such as globular clusters [1]. Interestingly, these types of interactions, called tidal disruption events (TDE's), are one of the only ways to directly observe black holes (BH's) due to how they affect the surrounding stars in the cluster. In the following chapters and sections, I intend to use data from hydrodynamic simulations to take a close look at the intricacies of these TDE's. My goal with this research is to unravel some of the mystery surrounding star-BH interactions inside of these dense star clusters.

Another important focus of this research is the analysis of the possible stellar remnant that can survive these TDE interactions. If the star passes within a certain distance from the center of the black hole, then a stellar remnant could survive the interaction; it is usually the star's core or inner layers that survive while the diffuse outer layers that get stripped away contribute to the formation of an accretion disk around the BH. In cases where the stellar remnant is unbound or has gained a sufficiently long orbital period, I want to observe its possible evolution with the interesting new condition of its unusual element composition. Spectral analysis of stars with irregular elemental composition could point to possible tidal disruption in that star's past.

1.2 Prior Work

Much research has been done on the topic of star – BH interactions, particularly on tidal disruption events within dense star clusters. In this respect, I will give a brief rundown on the main research papers from which I have drawn inspiration and information for this research project in this section. First, Kiroğlu et al. 2023 [2] focus on partial tidal disruption events of a sun-like main sequence star with an intermediate mass BH. In their research, the $1 M_{\odot}$ star that they use for the simulations was modeled with MESA (Modules for Experiments in Stellar Astrophysics, which will be covered in the Methodology section of this paper) which is the same stellar evolution code that I will be using to model the supergiant star I intend to use in my

hydrodynamic BH interactions [3]. It's worth noting that an intermediate mass BH (IMBH) is an object that lies between the two other classifications of BH, stellar BH's and SMBH's. IMBH's considered in their research range from $100\text{--}10^6 M_{\odot}$ ($1 M_{\odot}$ ("solar mass") is the mass of the sun or $1.99 \times 10^{30} kg$) and the resulting mass loss of their main sequence star was dependent on the strength of the interaction, i.e. the mass of the IMBH. They found that higher mass IMBH's created interactions where the star would lose a great amount of mass on the first passage at periapsis and the resulting star remnant would become unbound after only a few other passages. On the contrary, lower mass IMBH's created interactions where the initial mass loss from the star was less extreme, and the stellar remnant would remain bound to the BH to return for many more periapsis flybys; each flyby being associated with more mass loss. Regardless of IMBH mass and the number of pericenter passages, however, the stellar remnant would eventually become unbound and escape out to infinity in all cases.

The next research paper we will look at is Kremer et al. 2023 [4], which investigates tidal disruption of stars by stellar mass BH's within star clusters with a focus on mass accretion onto the BH. Using data from previous hydrodynamic simulations of tidal disruption events, the authors of this paper were able to create their own BH and accretion disk models from these previous runs in the effort to analyze how the matter in the accretion disk evolves. The matter that comprises the accretion disk was seen to contribute to the mass of the BH via mass fallback rates, stay in orbit around the BH within the accretion disk, or contribute to mass loss from the accretion disk from winds associated with Super-Eddington flow. Accretion disks around BH's in the context of this research are comprised of the diffuse material stripped from a star in TDE's, and this gravitationally bound material usually orbits the BH at extremely high velocities. With all of this material orbiting the BH within very close proximity and at such high velocities, the friction forces and loss of gravitational potential energy from falling material creates a tremendous amount of heat and electromagnetic radiation, which is why BH accretion disks are so luminous [5]. The authors show that when radiation from the accretion disk surpasses the Eddington Limit (or Eddington Luminosity), mass loss occurs via matter getting ejected from the disk. They expect that this is associated with the observation of bright transients within dense star clusters.

In Kremer et al. 2021 [6], TDE's between main sequence stars and BH's are analyzed on a cluster level, particularly that of a young star cluster. Stellar mass black holes are expected to be numerous within star associations and clusters of all masses based on observational data and this paper goes in depth on how the presence of BH's such as these can affect the evolution of the entire cluster. In a range of over 3,000 N-body simulations, they kept note of the cluster's mass, half-mass radius, and metallicity, finding that TDE's between stellar mass BH's and main sequence stars happened at a rate close to 200 per cubic gigaparsec per year. They conclude from the energy emitted from these events that they have much in common with Fast Blue Optical Transients, of which more observations are being made (e.g. Drout et al. [7]).

The last paper that I want to mention in this section is Kiroğlu et al. 2024 [7] which focuses on the effects of close encounters or collisions with stars on the properties of binary black holes (BBH's) inside dense star clusters. They found with Cluster Monte Carlo (CMC) N-body simulations that a large portion of the BH's that form within star clusters eventually interact with stars. BBH mergers are not exempt from this fact, with 10% to 60% of BBH mergers undergoing some form of stellar interaction in their data. Collisions such as these can affect the spin of these BBH's, associated with the dimensionless spin parameter X_{eff} . They also express how important models of young star clusters are since these environments are the most likely ones to produce collisions between BH's and massive stars. The particular importance of this paper to my research is the data from their Cluster Monte Carlo simulations. After communicating with the main authors of this paper, they informed me of the average values of important parameters such as star mass, radius, BH mass, the average timescale for collisions (about 7 million years), and that 95% of the giant stars undergoing interactions were helium burning in their core. The following figure from their paper illustrates these important parameters.

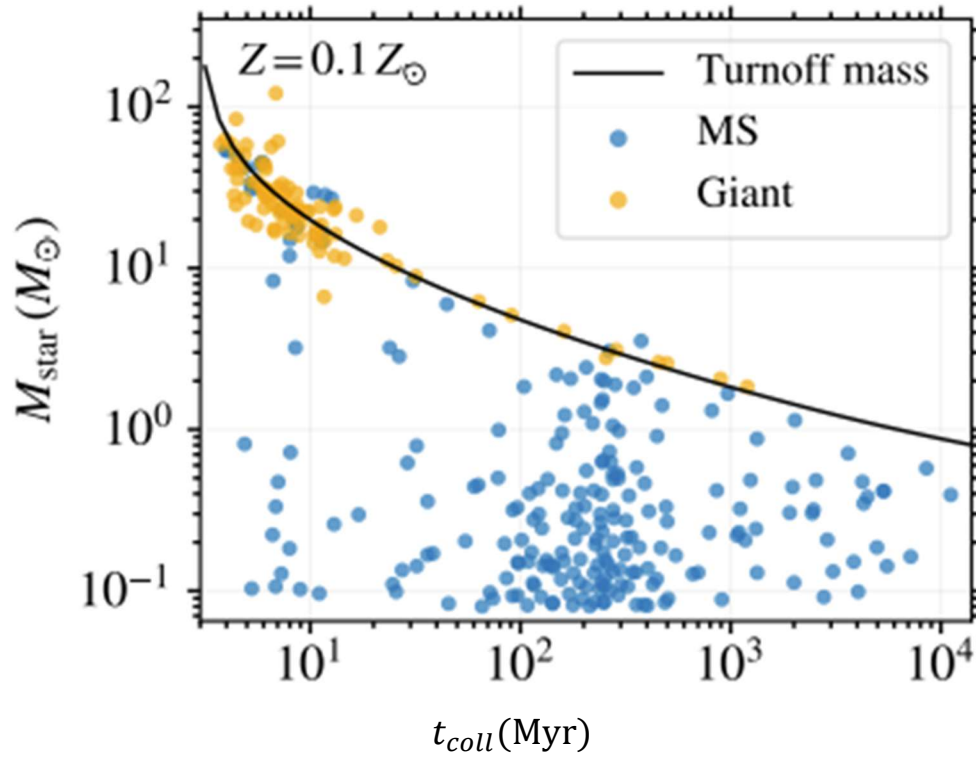


Figure 1: Figure 3 from Kiroğlu et al. 2024 [7]. This graph shows the star mass vs time of the collisions taken from CMC models 1-8. The orange dots represent giant star collisions and blue dots represent main sequence star collisions while the black line is the turnoff mass as a function of time. The turnoff mass of a star is the mass at which it begins turning into a giant star and turns off from the main sequence, which happens earlier for more massive stars and can be a good indicator for the age of a cluster.

It's worth noting from the above figure how in low metallicity, young stellar clusters, the earliest BH-star interactions that take place are with the more massive giant class stars. This figure and the helpful information that the authors sent me have led me to decide to use a $24 M_{\odot}$, helium core burning giant star model with a metallicity $Z = 0.002$ and a $30 M_{\odot}$ BH in my hydrodynamic simulations.

2 Theory

2.1 Gravity

The importance of the force of gravity cannot be understated. Despite being one of the weaker forces in the perspective of scale, it is the attractive force that is responsible for holding the entire universe in the structure that we find it in today. It is the phenomenon that allows galaxies, stars, and solar systems with planets to form, and it is responsible for the end of a star's life. Gravity's presence allows us to live our lives as we know them, and we would not be here without it. Our modern understanding of gravity was pioneered by Isaac Newton in the mid 1600's, and while we now have more accurate depictions of gravity from Albert Einstein's theory of General Relativity, Newtonian gravity has been shown to portray reality in many cases and is relevant to those who are learning physics and relevant to this project as well [8].

2.1.1 Newton's Laws of Motion

Isaac Newton is recognized across the globe today for creating some of the most foundational principles in classical mechanics and all physics. Some of his many contributions to the field are Newtonian Mechanics and his laws of motion first stated in his work, *Philosophiæ Naturalis Principia Mathematica* (or "The Principia") published in 1687 [9]. Newton's first law of motion brings forth the concept of inertia, that any given body that is in motion or at rest will stay in motion or at rest respectively unless acted upon by an outside force. The concept of a force that can act on objects is brought up in the second law of motion, which is of great importance to the whole of physics,

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}. \quad (2.1.1)$$

In the second law, $\vec{F}$ is the force acting on an object, m is the object's mass, and $\vec{a}$ is the resulting acceleration of the object, showing that the force that acts on an object is the product of the object's mass with its acceleration. The second law also shows how force is the time rate of change of an object's momentum $\vec{p}$. Equation (2.1.1) is foundational for describing the motion of objects and shows itself in many physical systems. One such system that will be important moving forward is the interior of stars. A star is a balance between internal pressure forces and

gravity, and the code that we use to do calculations for the interior of a star uses Newton's second law on each mass element that comprises it [10]. Newton's third law of motion focuses on the concept of equal and opposite reactions,

$$\vec{F}_{ab} = -\vec{F}_{ba}. \quad (2.1.2)$$

Here, $\vec{F}_{ab}$ is the force of object a acting on object b, and $\vec{F}_{ba}$ is obviously the force of object b acting on object a. Essentially, the third law of motion states how every action must have an equal and opposite reaction. An example or analogy that helped me understand the third law as a college freshman was that of a bus and a car having a head on collision; each object exerts the same force on each other, but the motion of the two objects after the collision is dependent on their individual masses. This also applies to the mass elements of a star model that I mentioned above, each mass element inside of the star model exerts forces on neighboring mass elements which exert an equal and opposite reaction force; these calculations done by the code are crucial in the process of relaxing star models [10].

2.1.2 Newton's Law of Universal Gravitation

I mentioned in the introduction to this section that Newton pioneered our modern understanding of gravity as an attracting force between objects at a distance. While his interpretation of the phenomenon was ultimately incorrect and Albert Einstein offered an improved model of gravity with his theory of General Relativity, Newtonian gravity still holds ground in many cases and is used frequently in the context of celestial objects who can physically interact with each other through gravity as a force. For the case of this project, the smoothed particle hydrodynamics (SPH) code *StarSmasher*, developed by Dr. James Lombardi Jr. and collaborators, uses Newtonian gravity to make calculations modeling the gravitational interactions between celestial objects [10]. In Newtonian gravity, the gravitational force $\vec{F}_G$ is proportional to the product of the masses of the two objects m_1 and m_2 divided by the square of the distance between them r . The gravitational constant is G and $\hat{r}$ is the unit vector that points away from the mass at the origin of our system [1],

$$\vec{F}_G = -G \frac{m_1 m_2}{r^2} \hat{r}, \quad G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}. \quad (2.1.3)$$

It is also worth noting that the gravitational force is negative, displaying that gravity is an attractive force that pulls objects to one another [1]. Gravity acts as a unifying force across the universe, and it governs the motion of galaxies, stars, and planets alike.

2.2 Elliptical Orbits

Before Newton published his law of universal gravitation in “The Principia” in 1687, a German astronomer named Johannes Kepler published two works that described the motion of the planets around the sun using geometry rather than physics [11]. While working under the astronomer Tycho Brahe, Kepler made observations of the orbit of the planet Mars that didn’t fit a perfectly circular orbit like they were expecting at the time. These observations led him to realize that Mars’s orbit was shaped like an ellipse, and hence that all orbits were ellipses rather than circular. He published his first two laws of planetary motion in 1609 in his work “*Astronomia Nova*” and later his third law in 1619 in his work “*Harmonices Mundi*” [11]. Despite not knowing about gravity at the time, Kepler’s laws of planetary motion are still reliable to describe the motion of orbiting bodies and were used by Newton while he was formulating his law of universal gravitation. The orbits of stars around black holes within dense star clusters are expected to be highly elliptical due to the chaotic environment inside these clusters, which is relevant to this research.

2.2.1 Some Geometry of Ellipses

An ellipse is characterized mainly by its two foci, its semi major axis, semi minor axis, and eccentricity [12].

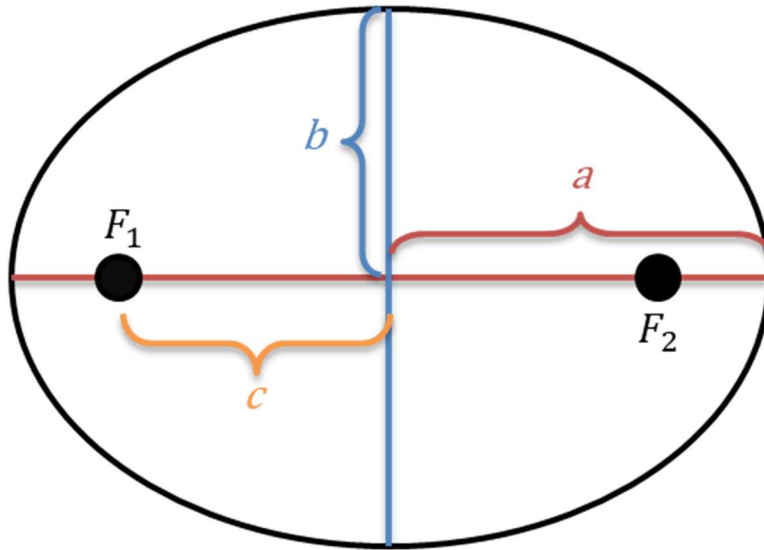


Figure 2: An ellipse with its defining qualities labeled. F_1 and F_2 are the two foci, a is the semi major axis, b is the semi minor axis, and c is the distance from one of the foci to the center of the ellipse.

As seen in Figure 2, the major axis $2a$ runs horizontally from one end of the shape to the other, intersecting each of the foci. The minor axis $2b$ runs vertically from the top co-vertex to the bottom co-vertex of the ellipse. In the following sections and equations, we only take half of the major and minor axes, the semi major and semi minor axes (a and b respectively), into account due to their importance. A crucial quality of ellipses is their eccentricity e ,

$$e = \frac{c}{a}. \quad (2.2.1)$$

The eccentricity of an ellipse is the ratio of the distance from the center to one of the foci to the semi major axis. One could think of the eccentricity as a measure of how “squashed” along the vertical axis the ellipse is, and its value in the range of zero to one and beyond tells you exactly what shape it is.

Eccentricity	Shape
$e = 0$	Circle
$0 < e < 1$	Ellipse
$e = 1$	Parabola
$e > 1$	Hyperbola

Table 1: The value of eccentricity can tell you exactly what shape you are dealing with, eccentricity of zero is a circle, between zero and one is an ellipse, equal to one is parabolic, and greater than one is hyperbolic [12].

Similar to how one could claim that a square is a special case of a rectangle where the sides are all of equal length, one could make the claim that a circle is a special case of an ellipse.

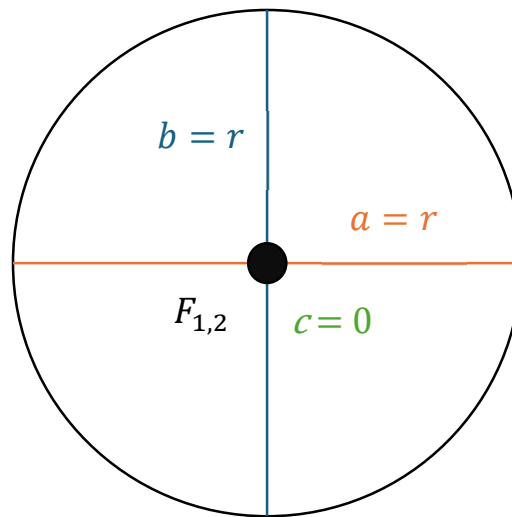


Figure 3: A circle is a special case of an ellipse where the distance between the center of the ellipse and the foci is zero and where the semi major and semi minor axes are equal.

As seen in Figure 3, the two foci of the ellipse have no distance between them ($c = 0$) and the semi major and semi minor axes are equal in length, making each of them equal to the radius of the circle. Since the distance c is equal to zero, we can infer from equation 2.2.1 that the eccentricity of a circle is in fact zero.

2.2.2 Elliptical Orbits and Kepler's Laws

Kepler's first law states that all orbits are ellipses with the center of mass of the two-body system placed at one of the foci of the ellipse [11]. Since the orbiting body is following an ellipse as its orbital path around one of the foci, this means that the distance between the orbiting body and the center body is never constant, so there must be some minimum and maximum distance between the two bodies. These two distances are the periapsis and apoapsis separations, respectively.

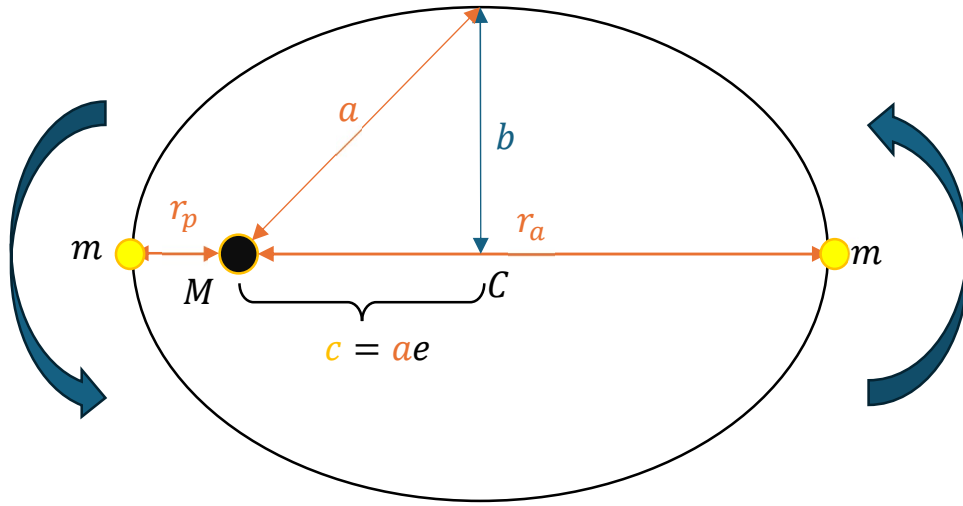


Figure 4: A diagram of the orbital path of a star m in an elliptical orbit around a BH M , the arrows indicate the direction of the angular velocity of the star in its orbit. The periapsis and apoapsis separations, r_p and r_a respectively are labeled as well. It's also worth noting that, due to the geometry of an ellipse, the distance between one of the foci and the co-vertex of the ellipse is always equal to the semimajor axis of the ellipse [12].

From Figure 4, we can derive the equations for the periapsis separation—the distance of closest approach—and the apoapsis separation—the farthest distance of the star from the BH,

$$r_p = a - ae = a(1 - e), \quad (2.2.2)$$

$$r_a = a + ae = a(1 + e), \quad (2.2.3)$$

$$c = ae. \quad (2.2.4)$$

It's worth noting that we obtained equation 2.2.4 by rearranging equation 2.2.1 to solve for the distance c , showing that c is always equal to the product of the semi major axis and the

eccentricity of the ellipse. The periapsis separation r_p is especially important for this research, if r_p gets within the tidal radius of the BH—which will be discussed later—then tidal disruption occurs.

Kepler’s second law states that an imaginary line connecting the center body and the orbiting body sweeps out equal areas in equal times [11]. This expresses the fact that the orbiting body’s velocity along its path is never constant, rather the orbiting body has the highest velocity at periapsis and the lowest velocity at apoapsis, which we can infer from conservation of angular momentum.

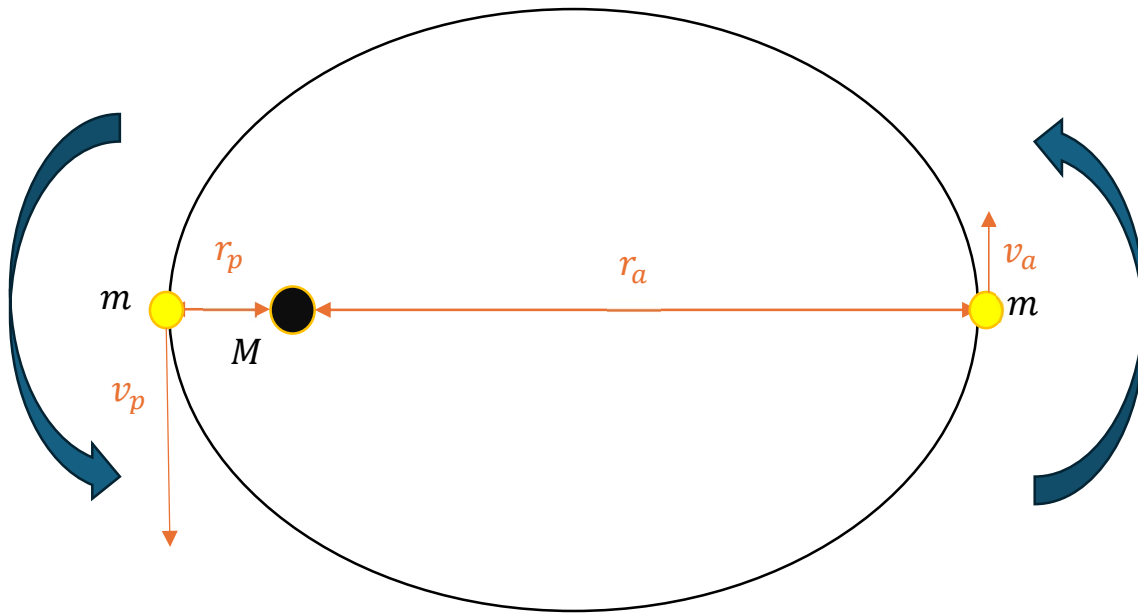


Figure 5: A diagram expressing the difference in velocity of the orbiting body at periapsis and apoapsis, v_p and v_a respectively. The star’s tangential velocity is much larger at periapsis than at apoapsis due to conservation of angular momentum.

If we want to know the star’s velocity at any point along its orbit, we can use the Vis-Viva equation. This equation is derived from considering the total energy and angular momentum at periapsis and apoapsis [12]. We can start by considering energy conservation across the orbit,

$$E_{tot} = E_p = E_a \Rightarrow \frac{v_p^2}{2} - \frac{GM}{r_p} = \frac{v_a^2}{2} - \frac{GM}{r_a}$$

$$\Rightarrow \frac{1}{2}(v_a^2 - v_p^2) = GM\left(\frac{1}{r_a} - \frac{1}{r_p}\right).$$

Next, we'll consider angular momentum across the whole orbit,

$$L_{tot} = L_p = L_a \Rightarrow v_p r_p = v_a r_a \Rightarrow v_p = \frac{r_a}{r_p} v_a.$$

Substituting in the expression for v_p from angular momentum conservation into the equation for energy conservation and utilizing the fact that the sum $r_p + r_a = 2a$ yields the expression for the total energy of the system:

$$E_{tot} = -\frac{GM}{2a}. \quad (2.2.5)$$

When plugging this expression into a general energy conservation equation for the whole orbit, we get

$$T + U = E_{tot} \Rightarrow \frac{v^2}{2} - \frac{GM}{r} = -\frac{GM}{2a}$$

and then solving for the velocity, we get

$$v = \sqrt{GM \left(\frac{2}{r} - \frac{1}{a} \right)}. \quad (2.2.6)$$

It is worth noting that the common factor of the mass of the star cancels out, showing that the orbiting body's mass is negligible if the mass of the BH is great enough.

Kepler's third law states that the square of the orbital period is proportional to the cube of the semi major axis [11],

$$T^2 \propto a^3. \quad (2.2.7)$$

This essentially means that the time to complete an orbit increases drastically when the semi major axis is large, like how the orbit of Mercury around the sun takes a much shorter amount of time than the orbit of Neptune, the farthest planet from the sun (sorry Pluto). I can apply this to

the second goal of my research; if the calculated orbital period of the star becomes greater than or equal to the necessary timescale for it to contract back to a normal size, then I can use the simulation in my results.

2.3 Stars

Stars are the lighthouses of the universe and are the beating heart of this research. Stars illuminate and make the presence of their respective islands (solar systems) known throughout the vast, dark ocean of space. Their sheer numbers throughout the observable universe know no bounds, with $\sim 10^{22}$ stars occupying various star clusters and galaxies as they permeate throughout the cosmos [13]. Stars exist as a constant battle between nuclear forces and the weight of gravity. They function almost like engines that cook lighter elements into heavier ones, and they are responsible for seeding the universe with the heavy elements necessary for planets like Earth with life to take form. Were it not for the generations of stars before our time, we would not be here right now to research them. The following sections will briefly explore the lives of stars and how they have come to exist in the universe, as well as the importance of creating accurate models of them in the relevant code.

2.3.1 Hydrostatic Equilibrium

As I mentioned in the introduction of this section, a star is physically a battle between the internal pressure created by nuclear fusion and the weight of the star itself caused by the force of gravity trying to collapse the star's structure into itself. With this in mind, there are three defining characteristics of a star that are responsible for its structure and spheroid appearance: density, pressure, and hydrostatic equilibrium [13]. Density in general is defined as an amount of mass, a measure of matter, in a certain volume or amount of space. The average density of a star would be expressed mathematically as a simple ratio,

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}. \quad (2.3.1)$$

Here, the average density is determined by its mass M over its volume, which we can see is just geometrical equation for the volume of a sphere, with the value of R being the radius of the star. However, in the following equations, the average density of the star will not accurately represent

other values since the density of the star changes depending on the position away from the center that we consider, as evident from the following figure.

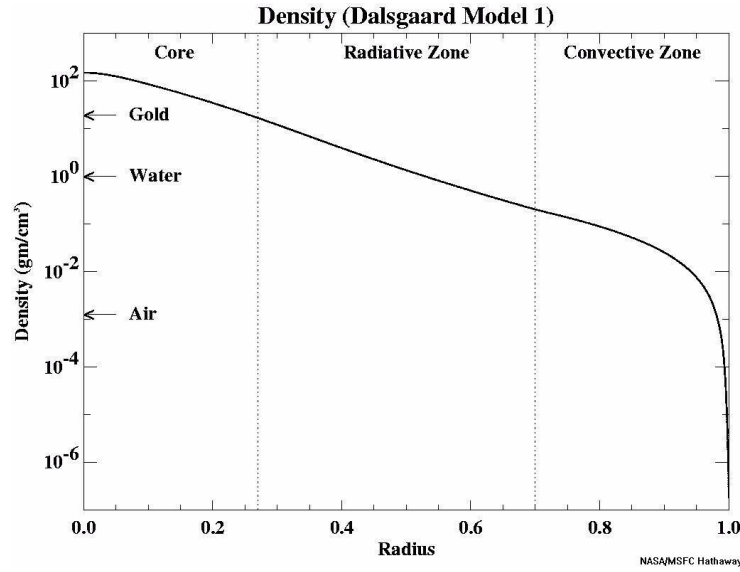


Figure 6: The density profile of the sun, we can see that the density of plasma decreases gradually as we travel away from the core [15].

A star is comprised of plasma, an extremely hot gaseous substance where the temperature is so extreme that electrons are no longer bound to atoms, forming a superheated ionized state of matter [14]. This extreme composition gives stars a great amount of internal pressure that can counteract the massive weight of these objects. For low mass stars, the internal pressure can be expressed as

$$P = \frac{\rho k T}{\mu m_p}, \quad k = 1.381 \times 10^{-23} \text{ J/K.} \quad (2.3.2)$$

Here, ρ is the density gradient function of the star $\rho(r)$, k is Boltzmann's constant, T is the internal temperature, and m_p is the mass of a proton. The mean molecular mass μ can be thought of as the average number of atomic mass units (amu's) per particle; in this case, per proton [15].

The internal pressure that a star exerts outward is vital for it to maintain its spheroid shape, lest it collapse in on itself from its sheer weight under the force of gravity. The condition for when the star has found a balance between its own gravity and its internal gas pressure is called hydrostatic equilibrium and is the reason why massive objects in space are usually

spherical in shape. Under this condition, the star “relaxes” to a radius that equalizes these two forces.

$$\frac{dP}{dr} = -G \frac{M_r \rho}{r^2} = -\rho g \quad (2.3.3)$$

Here, $\frac{dP}{dr}$ is the pressure gradient from the interior of the star, M_r is the mass enclosed in some radius r , and ρ is again the density function from equation 2.3.2. The ratio $\frac{M_r \rho}{r^2}$ is equal to the gravitational acceleration at a distance r from the center of the star, similar to how $g = 9.8 \frac{\text{m}}{\text{s}^2}$ on the surface of Earth. When the pressure gradient is equal to the force of gravity at the surface of the star, then hydrostatic equilibrium is met, and the star rests at some constant radius for the time being. This phenomenon is pictured in the following figure.

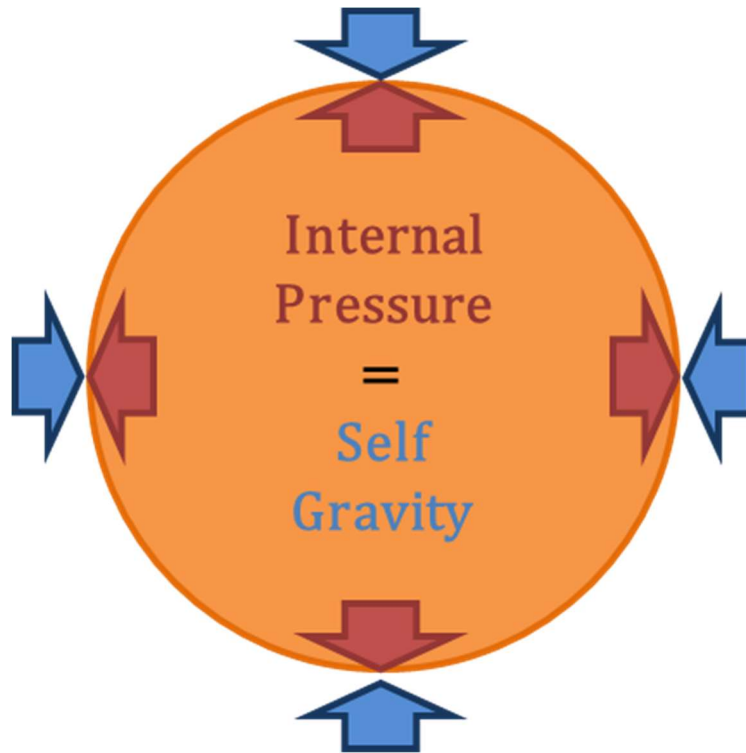


Figure 7: The condition for hydrostatic equilibrium. The gradient of the internal gas pressure must equal the star’s own gravity for the star to be static. Otherwise, it would collapse under its own weight or explode out into space.

2.3.2 Stellar Evolution

From the moment a star is born, its death has already been predetermined by factors such as its mass and composition. Massive stars burn brilliantly throughout their short lives and die spectacularly in luminous supernova explosions, whereas less massive stars burn dimmer for a much longer period before dying out. Everything that happens between the birth and death of a star is part of its evolution; a star changes drastically in regard to its size and composition from its beginning to its end. The driving force behind this evolution is nucleosynthesis via nuclear fusion.

Stars are born from the gravitational collapse of interstellar clouds of dust and gas, usually comprised of mostly molecular hydrogen. Over time, the cloud accumulates into a denser region as gravity pulls more material in towards the center. As more and more of the cloud falls into the center, the region begins to increase in temperature, forming a protostar. The protostar continues to heat up as it continues pulling matter into itself, adding to its mass and moving towards the main sequence of the Hertzsprung-Russell Diagram (HR diagram, shown in Figure 7). Once the protostar has accumulated enough mass, its interior gets to such an extreme temperature that hydrogen nuclei have enough energy to overcome their mutual Coulomb repulsion as the nuclear strong force makes them bind to each other to form helium nuclei, a process called nuclear fusion [16]. When hydrogen burning has been achieved in the core of the newly born star, it has made it to the main sequence of the HR diagram and is now a main sequence star, where it will continue burning hydrogen into helium for most of its life, possibly billions of years.

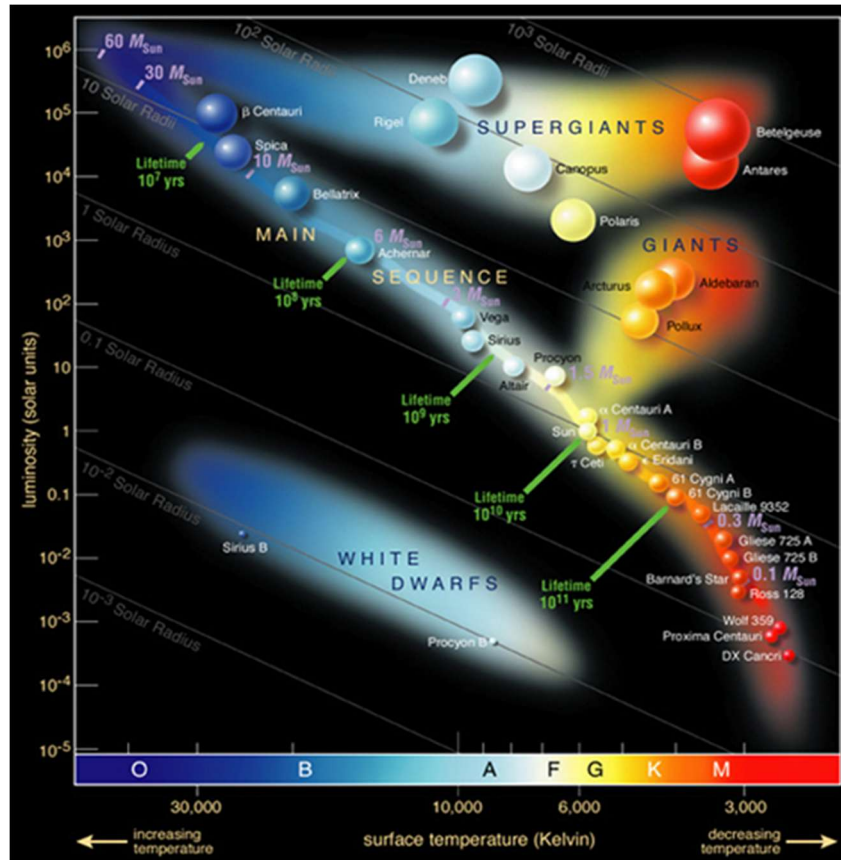


Figure 8: The Hertzsprung-Russell Diagram, a way of cataloguing stars based on their luminosity and effective temperature [17]. Something important to note about the HR diagram is how the effective temperature axis increases in value in the left direction rather than the right. The line of stars that runs from the top left down to the bottom right is the main sequence line, where most stars are observed to be since they spend a vast amount of time as main sequence stars. Amidst the main sequence stars, the sun can be found since it is a middle-aged main sequence star itself. In the top right of the graph, the larger giants and supergiants can be found.

When the star has burned all the hydrogen in its core into helium, a shell of hydrogen burning forms around the new helium core. This new increase in energy output from the hydrogen burning shell causes the outer layers of the star to expand drastically, making its surface temperature decrease but its luminosity increases due to the increase in surface area. As this happens, the star moves across the Hertzsprung gap to the giant branch of the HR diagram; a bell tolls for the star as this is a signifier for the near conclusion of its life. Along the giant branch, the star continues to fuse heavier and heavier elements in its core and subsequent shells surrounding the core via nuclear fusion. This process repeats, from hydrogen to helium, helium to carbon, carbon to neon, neon to oxygen, oxygen to silicon, and finally silicon to iron. The final element that the star reaches is dependent on the mass of the star. Due to the binding energy per

nucleon of iron, it cannot be fused into heavier elements without energy being added to the system, creating a core of dead iron ash (more on this in the following section).

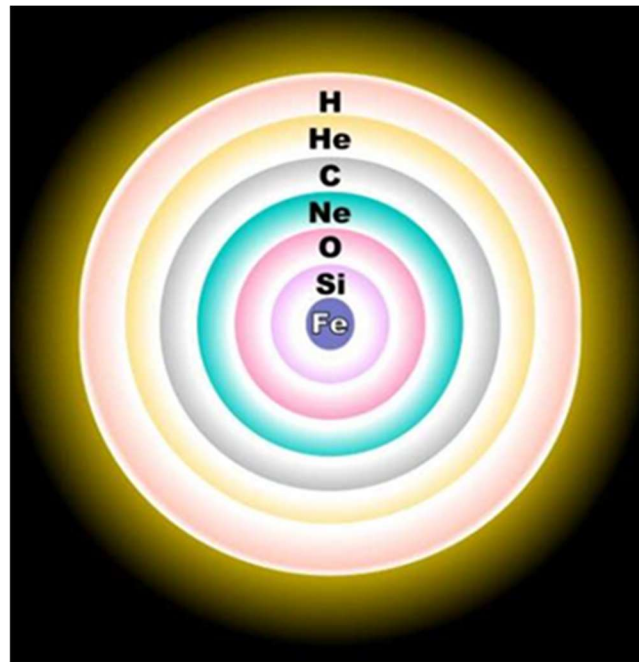


Figure 9: The shells of element burning that form near the end of a massive giant star's life, resembling an onion [18]. Since iron has been achieved within the core, the star doesn't have much longer before its core collapses and the star explodes in a supernova.

Without the internal radiation pressure from fusion in the core of the massive star, the battle between internal pressure and gravity is over, and gravity is victorious. Hydrostatic equilibrium no longer applies to the star as the outer layers begin to fall towards the core. They collapse and then rebound against each other, propelling all the material that made up the layers of the star out into space in an enormous, luminous, violent supernova explosion. In its death, the giant star spews its guts out into the galaxy, seeding the surrounding area with heavy elements that will no doubt find themselves within new generations of stars.

What's left of the star is its core, which could either be a neutron star or a black hole depending on the star's mass. If the star's core is less than $3 M_{\odot}$, then its collapse will be halted by neutron degeneracy pressure and form a neutron star. If the star's core is greater than $3 M_{\odot}$, then nothing can stop its collapse as it continues to fall into itself and form a stellar mass black hole.

2.3.3 Stellar Nucleosynthesis

There are two main ways that stars undergo hydrogen fusion in their interiors, the proton-proton chain (p-p chain) and the carbon-nitrogen-oxygen cycle (CNO cycle). The p-p chain is more prevalent in less massive stars like the sun as they exist on the main sequence and involves the fusion of hydrogen into deuterium, deuterium and hydrogen into helium 3, and ultimately helium 3 into helium 4 and hydrogen nuclei along with energy.

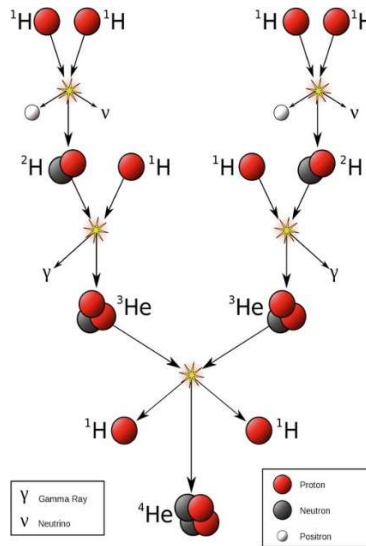


Figure 10: A diagram detailing the steps of the p-p chain [19]. Two protons fuse to create deuterium, a positron, and a neutrino. The deuterium fuses again with a proton to create helium 3 and a gamma ray. Two helium 3 nuclei fuse to create helium 4 and two protons.

The CNO cycle, on the other hand, is more common within higher mass or higher metallicity main sequence stars. The process involves a circle of reactions, starting with carbon 12 fusing with a proton to make nitrogen 13 and ending with another carbon 12 atom alongside a helium 4 nucleus.

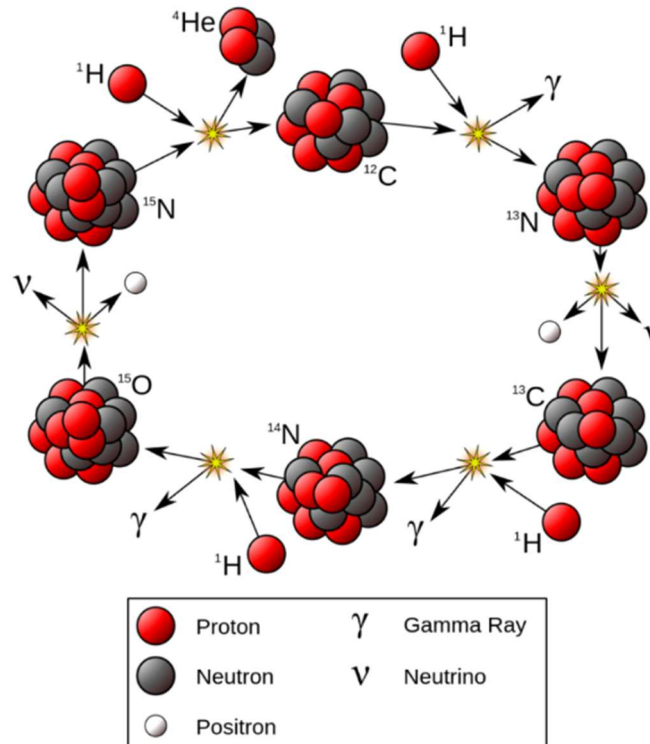


Figure 11: A diagram of the CNO cycle [20]. The process begins with a carbon 12 nucleus fusing with a hydrogen nucleus to create nitrogen 13 and a gamma ray. Since the half life of nitrogen 13 is not very long, it decays into carbon 13, a positron, and a neutrino. The carbon 13 nucleus then fuses with a hydrogen nucleus to create nitrogen 14 and a gamma ray; the nitrogen 14 nucleus then fuses with a hydrogen nucleus to create oxygen 15 and a gamma ray. The oxygen 15 nucleus eventually decays into nitrogen 15, a neutrino, and a positron. The nitrogen 15 nucleus then fuses with a hydrogen nucleus to create helium 4 and carbon 12, which starts the entire process over again.

Once a star has exhausted all the hydrogen in its core, it begins fusing helium into carbon via the triple alpha process. This reaction involves two alpha particles (helium 4 nuclei) fusing into an unstable isotope beryllium 8. If a third alpha particle fuses with the beryllium nucleus before it can decay, then a carbon 12 nucleus is formed.

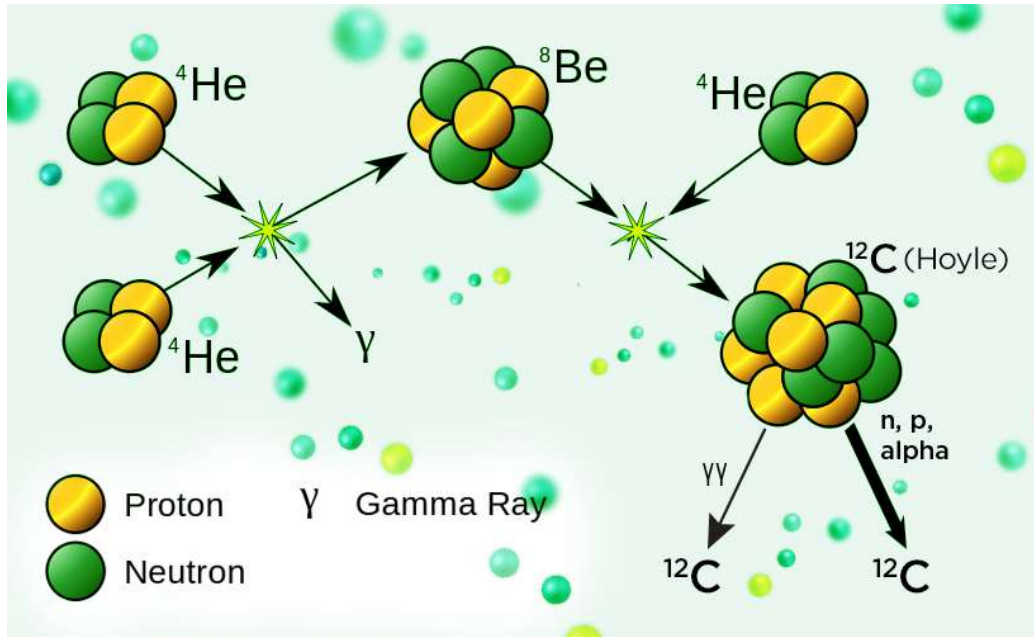


Figure 12: A diagram of the triple alpha process [21]. This reaction gets its name from the involvement of three alpha particles (helium nuclei). Two of which come together to form beryllium 8 and a gamma ray, then the beryllium fuses with the third alpha particle to form carbon 12.

In the prior section on stellar evolution, I mentioned how giant stars cannot fuse elements heavier than iron in their cores. The reason for this is dependent on the binding energy per nucleon (BEN) of the isotope of iron that gets created within stars. In simple terms, the BEN is the amount of energy needed to separate a nucleon, such as protons or neutrons, from the rest of the nucleus. It just so happens that iron isotopes are the most stable forms of nuclei in nature, meaning that iron has the highest BEN of any element, even heavier ones. Because of this, fusing iron with other nuclei to form heavier elements requires an input of energy, rather than an output of energy which is required for the case of hydrostatic equilibrium within giant stars; hence why stars undergo supernova when their core has reached iron.

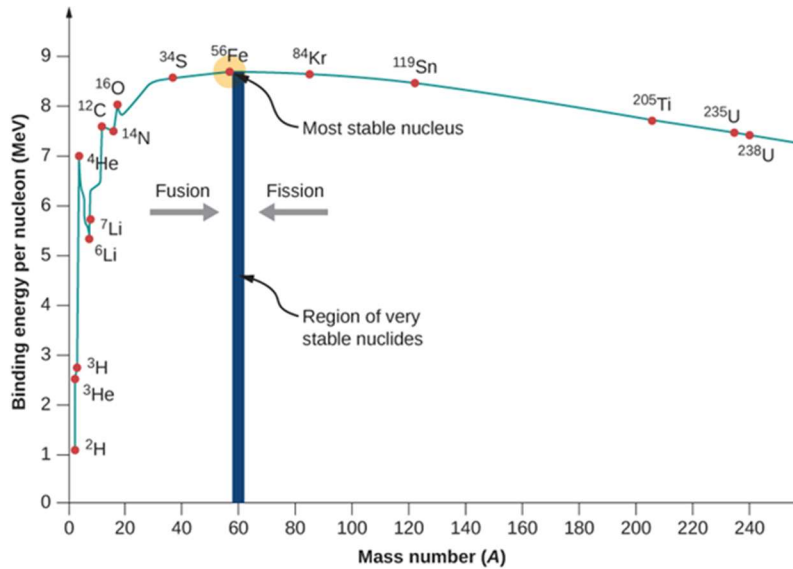


Figure 13: Graph of BEN vs mass number of various element isotopes [22]. Starting from the left side of the graph, where the light isotopes are, traveling up the curve from one element to the next corresponds to the amount of energy that is released in the fusion reaction on the BEN axis. Since iron 56 is the peak of the curve, going to the next element corresponds to a decrease on the BEN axis, signifying that an energy input is necessary for the reaction.

2.4 Black Holes

Of all the interstellar objects that we have observed over the course of history, perhaps none are as bizarre or as mysterious as the elusive black hole. Most of the BH's in the observable universe are stellar mass BH's, the corpses of once massive, giant stars and is what my research will focus on. There are two other categories of BH's however, intermediate mass BH's (IMBH) have a mass range of $100-10^6 M_{\odot}$ and supermassive BH's (SMBH), truly enormous BH's with masses reaching that of millions or even billions of suns which usually occupy the center of galaxies. BH's are notoriously difficult to observe since they emit very little electromagnetic radiation. One of the few ways we have of observing them is observing how they affect their surrounding stellar neighbors within dense star clusters, for example. Because of the lack of observable features of BH's, there are only three physical properties we can use to describe them: mass, charge, and spin, very similar to subatomic particles. One could make the claim that BH's are like really, really massive electrons, for example. BH's remain as a hot topic in the field of astrophysics because of how inherently mysterious they are; perhaps not until the distant future, we may never know exactly what lies beyond the event horizon of a BH.

2.4.1 The Schwarzschild Radius

In the introduction to this section, I mentioned something called the “event horizon” of a BH. The event horizon is a threshold, a distance from the center of the BH where the velocity you would need to escape its gravitational pull becomes the speed of light itself. Because of this, nothing in the universe, including light, can escape the BH’s pull, outlining the edge or barrier of the BH beyond which we cannot observe and giving the BH its void-like appearance. The radius away from the center of the BH that defines the event horizon is called the Schwarzschild Radius, named after the German astronomer Karl Schwarzschild, who calculated this solution for Einstein’s theory of General Relativity in 1916 [23]. Due to the presence of general relativity here, there is implied time dilation around a BH. In fact, the theory concludes that light at the event horizon is frozen in time due to the immense gravity.

While general relativity is necessary for an accurate derivation of the Schwarzschild radius, you can still get the correct equation for it by using the simple case of energy conservation at infinity, by sheer coincidence. We start by considering that the total energy at infinity is equal to zero,

$$T + U = E_{\infty}, \quad E_{\infty} = 0 \quad (2.4.1)$$

$$\Rightarrow \frac{1}{2}mv_{esc}^2 - \frac{GMm}{r} = 0 \Rightarrow \frac{1}{2}v_{esc}^2 = \frac{GM}{r}.$$

Next, we consider the escape velocity v_{esc} to be the speed of light c and then solve for r ,

$$r_s = \frac{2GM}{c^2}. \quad (2.4.2)$$

Finally, we are left with the equation for the Schwarzschild Radius r_s which is proportional to the mass of the black hole M divided by the speed of light squared c^2 , showing the condition that the escape velocity is equal to the speed of light at the event horizon of a BH.

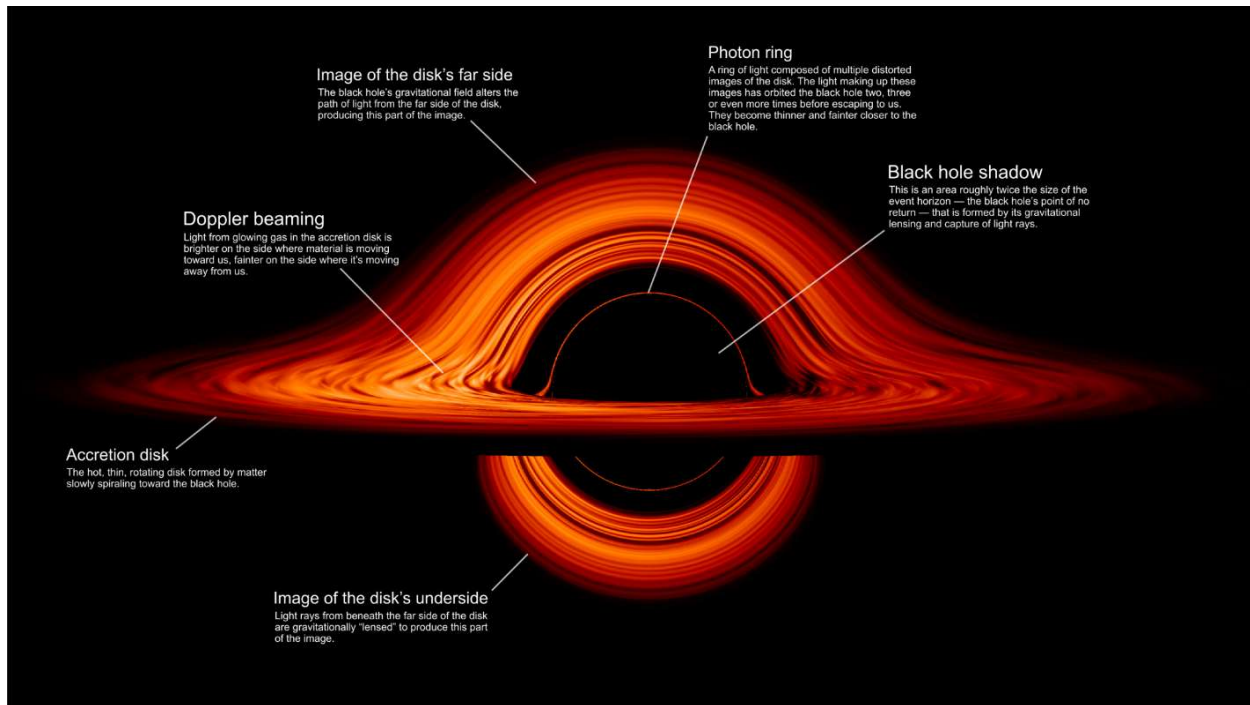


Figure 14: Diagram of a BH from NASA's Goddard Spaceflight Center [24]. The basic structure is the BH in the center, visually marked by the event horizon with an orbiting accretion disk of matter. Since gravity can change the path of light, you can see the part of the accretion disk that is behind the BH above and below it.

2.5 Tidal Disruption Events

Within a dense stellar cluster such as a globular cluster, you are bound to find a decently sized population of stellar mass BH's if the cluster is the right age. With the presence of these BH's in the cluster, odds are that some of these BH's will undergo some form of gravitational interaction with their neighboring stars. The most common observed interaction between these two objects is a star that is bound to the BH in a highly elliptical orbit. In cases like these, if the star gets sufficiently close to the BH—with a small enough periapsis separation—the star could undergo tidal disruption. Tidal disruption falls under two different types of events, tidal disruption events (TDE's) and partial tidal disruption events (PTDE's). In the former, the star gets too close within the BH's tidal radius and gets completely destroyed by tidal forces. In the latter, the star passes close enough to or within the BH's tidal radius to get part of its mass stripped away but not entirely destroyed, the remnant of the star could continue orbiting the BH or it could escape out into space while the stripped mass contributes to the accretion disk of the BH.

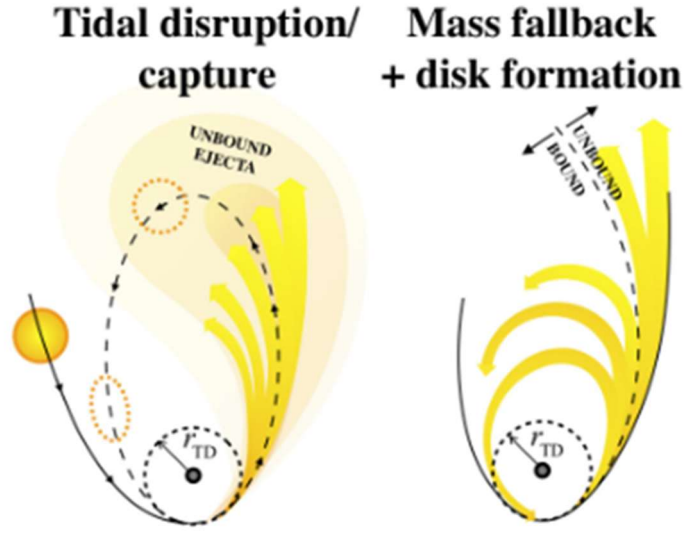


Figure 15: Part of an illustration of a tidal disruption event from Kremer et al. 2021 [6]. The star in the figure's pericenter passage is within the tidal radius of the BH and it is ultimately destroyed.

2.5.1 Tidal Forces and the Tidal Radius

The driving force behind TDE's and PTDE's is the BH's tidal force, which is caused by a difference in gravitational force at two different points on the star as it passes through its periastris distance. At a far enough distance, this merely causes a bulging of the star in the direction of the BH, much like the tides on Earth caused by the moon; but at a close enough distance, tidal disruption occurs. Using Newton's law of universal gravitation from equation 2.1.3, we can use the difference between the force of gravity acting on a mass element m on the closest point on the surface of the star to the BH and an equal mass element m at the center of the star to derive the equation for the tidal force,

$$\vec{F}_c = -G \frac{M_{BH}m}{r^2} \hat{r}, \quad (2.5.1)$$

$$\vec{F}_s = -G \frac{M_{BH}m}{(r-R_*)^2} \hat{r}. \quad (2.5.2)$$

Subtracting equations 2.5.1 and 2.5.2 yields

$$\vec{F}_T = \vec{F}_s - \vec{F}_c = GM_{BH}m \left(\frac{1}{(r-R_*)^2} - \frac{1}{r^2} \right) \hat{r}.$$

From here, we can use a Taylor expansion to approximate the term on the right to obtain the tidal force of the BH,

$$\vec{F}_T \approx \frac{2GM_{BH}mR_*}{r^3} \hat{r}, \quad (2.5.3)$$

where G is the Newtonian gravitational constant from equation 2.1.3, M_{BH} is the mass of the BH, m and R_* are the mass of the mass element and radius of the star respectively, and r is the distance between the star and the BH. All these quantities and the case derived above are shown in the following figure.

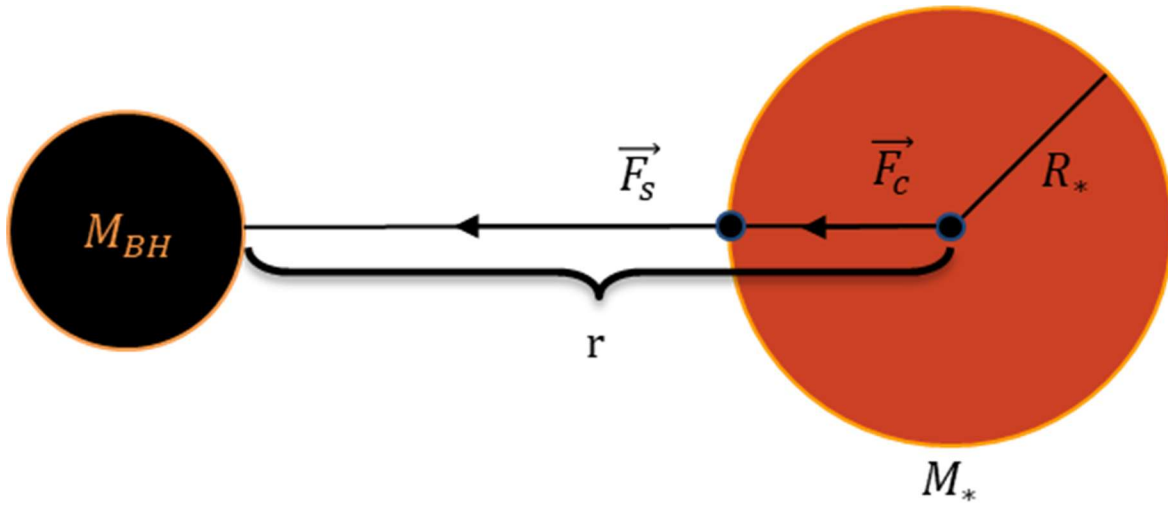


Figure 16: A diagram demonstrating the quantities that are important for the derivation of equation 2.5.3. A star (right) is pictured getting close enough to a BH (left) for there to be a significant difference in the gravitational pull that the BH exerts on the surface of the star compared to the gravitational pull on the center of the star, resulting in a tidal disruption event.

An important consequence of the tidal force of the BH is the tidal radius, which is located where the tidal force of the BH is equal to the star's internal force of gravity that maintains its shape. Setting these two forces equal to each other and solving for r will give us the equation for the tidal radius of the BH,

$$\begin{aligned} F_T = F_g &\Rightarrow \frac{2GM_{BH}mR_*}{r^3} = G \frac{M_*m}{R_*^2} \\ &\Rightarrow r^3 = 2R_*^2 \frac{M_{BH}}{M_*} \end{aligned}$$

$$r_t = R_* \left(\frac{M_{BH}}{M_*} \right)^{1/3}. \quad (2.5.4)$$

Thus, we can see from equation 2.5.4 that the tidal radius is proportional to the cube root of the ratio of the BH's mass over the star's mass multiplied by the radius of the star.

2.6 Energy Lost to Tides and “Kicking” a Remnant

Something important to consider when dealing with gravitational interactions between stars and black holes is the portion of orbital energy that contributes to the tidal deformation of the star and how that can affect its orbital path around the black hole. This concept is explored in depth in the paper Kremer et al. 2022 [25] which used mathematical results from the paper Press & Teukolsky 1977 [26]. The orbital dynamics of the simulations they carried out in their paper were affected by the amount of total energy in the orbit that contributed to oscillation modes of the star through tidal forces. This phenomenon is responsible for the formation of compact binary star systems through tidal capturing [26].

In the event of a star encountering a black hole, the total orbital energy can be expressed as

$$E_{orb} = E_{tides} + E_{bind} + E_{kick} \quad (2.6.1)$$

$$= - \frac{GM_* M_{BH}}{2a}. \quad (2.6.2)$$

In the above equation, total orbital energy is the sum of the energy contributing to tides (E_{tides}), binding energy of the star and any ejected material (E_{bind}), and the kinetic energy of the surviving star remnant post encounter (E_{kick}). The energy injected into oscillating the star is given by

$$E_{tides} \approx - \left(\frac{M_{BH}}{M_*} \right)^2 \left(\frac{GM_*^2}{R_*} \right) \left[\left(\frac{R_*}{r_p} \right)^6 T_2 + \left(\frac{R_*}{r_p} \right)^8 T_3 \right]. \quad (2.6.3)$$

Here, T_2 and T_3 are dimensionless functions associated with the oscillation modes of the star. This equation is a result from Press & Teukolsky, which does not take mass strippage of the star into account, but we can still make a loose usage of it in the discussion of simulation results later [26]. There are many more terms in the above summation, but we need only consider the first term for the purposes of this project. Because E_{tides} and E_{bind} are both negative, if $E_{kick} <$

$|E_{tides} + E_{bind}|$, then the total orbital energy becomes negative and the stellar remnant is bound to the black hole [25]. If the opposite is true ($E_{kick} > |E_{tides} + E_{bind}|$) then the resulting remnant would have a greater kick velocity and would become unbound, although this is unlikely to happen unless the mass stripped from the star is equal to or exceeds that of the surviving remnant. In which case, the asymmetry created in the tidal tails of stripped matter from the star impart kinetic energy to the remnant, “kicking” it out of the system with a high velocity [2]. The kick velocity can be approximated from an equation for E_{kick} taken from Kremer et al. 2022 [25],

$$E_{kick} \approx \frac{GM_{BH}\Delta M}{r_T^3} R_*^2 = \frac{1}{2} M_{rem} v_{kick}^2 \quad (2.6.4)$$

$$\Rightarrow v_{kick} \approx \left(\frac{2GM_{BH}\Delta M}{M_{rem}r_T^3} \right)^{\frac{1}{2}} R_*. \quad (2.6.5)$$

Where M_{BH} is the black hole mass, ΔM is the mass loss from the star, G is the gravitational constant from equation (2.1.3), r_T is the tidal radius of the black hole, and R_* is the radius of the star. This approximation really holds only for PTDE’s, since it implies that $v_{kick} \rightarrow \infty$ as $M_{rem} \rightarrow 0$.

In the case of a bound stellar remnant, this process of injecting orbital energy into tides would extend through each subsequent pericenter passage, resulting in a continuously lowering orbital period (and shrinking semimajor axis) until the star is possibly completely disrupted by the black hole. Upon the initial encounter between a star and a black hole in a star cluster, the star’s entry trajectory is usually a parabolic orbit toward the black hole, which is associated with an initial semimajor axis $a = \infty$ and an orbital energy $E_{orb} = 0$. After the first encounter, E_{orb} becomes equal to E_{tides} due to the tidal oscillations of the star (now $a < \infty$). Using this logic to equate equations (2.6.2) and (2.6.3), we can see the semimajor axis of the new orbit is proportional to r_p^6 . Applying this relation to Kepler’s third law, which scales the orbital period $P \sim a^{\frac{3}{2}}$, we can see that $a^{\frac{3}{2}} \sim r_p^9$. This means that plotting orbital period with periapsis on a log-log scale would show a slope approaching nine at greater periapsis values, which is a result discussed in Press & Teukolsky [26]. This relationship will become important in later analysis sections of this paper.

3 Methodology

Earlier in this paper in section 1.2 when I reviewed the prior work done in this research area, the last paper that I discussed was particularly important to my research project. In the paper, Kiroğlu et al. 2024 [7], the authors conducted research involving the use of Cluster Monte Carlo simulations of young stellar clusters. Cluster Monte Carlo code is a way to efficiently model star clusters by employing an N-body treatment (dynamical treatment of particles) over the entire cluster and keeping track of interactions that some bodies have with their neighbors [1] [27]. Modeling dense star clusters like this is much less computationally expensive and allows you to gather information about exactly what types of objects undergo collisions or interactions at a certain age of the cluster. After getting in contact with the authors of the paper and making a note of figure 3 (figure 1 in this paper), they gave me important parameters such as the average giant star mass ($24 M_{\odot}$), radius ($\sim 20 R_{\odot}$), metallicity ($Z = 0.002$), BH mass ($30 M_{\odot}$), timescale for collisions (~ 7 Myr), and the fact that 95% of the giant stars in their CMC data were helium core burning. Hence my decision to use a $24 M_{\odot}$ blue supergiant star model with a radius of $25 R_{\odot}$ ($1 R_{\odot}$ is the radius of the sun or 6.96×10^8 m) with a $30 M_{\odot}$ stellar BH in most of my collision simulations.

3.1 Modules for Experiments in Stellar Astrophysics

The stellar evolution code that we use to make the models of our stars is Modules for Experiments in Stellar Astrophysics, or MESA, developed mainly by Bill Paxton and various computational astrophysicists over a ten-year period and is now upheld by an international team [28]. MESA can simulate the life of a star as a path along the HR diagram depending on the initial inputs given. Depending on parameters like the star's mass, metallicity, etc., the code can show you where its life begins, where it ends, and important information about the star itself along the way. The code creates a LOGS directory that contains various "profile" files of the star, each of which contain information such as its composition, photosphere radius, age, the core element burning stage it is at, and much more about the star at the associated point in time along its path. To fit the CMC data that the authors of the aforementioned paper sent me, I told MESA to start a run of a star of $24 M_{\odot}$ and a metallicity of $Z = 0.002$. Once the run had finished, I looked through the profile files in the LOGS directory and settled on one that contained a

photosphere radius value as close to $20 R_{\odot}$ as possible. This ended up being profile669.data, with a radius value of a little over $25 R_{\odot}$ and a helium core mass of $9.8 M_{\odot}$ with an increasing value for center carbon 12 showing that it was fusing helium into carbon.

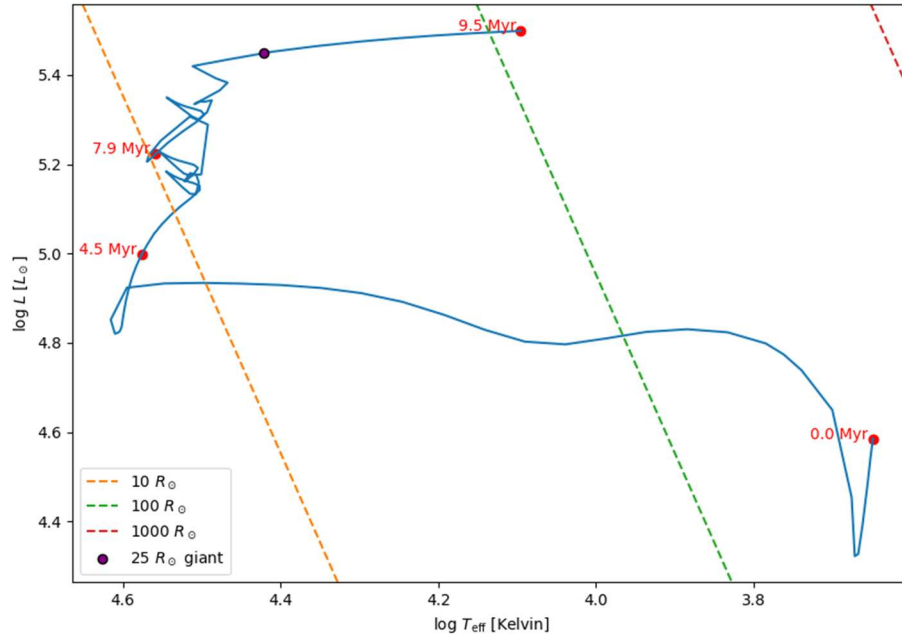


Figure 17: The HR diagram of my $24 M_{\odot}$ supergiant star MESA run. The star's life is outlined by the blue line with timestamps labeled in red. The colored dashed lines represent where a star of a certain size would be located, such as 10, 100, and $1000 R_{\odot}$. The star begins its life on the far-right side of the path as a cloud of gas, labeled 0.0 Myr, as it continues to collapse down and travel over to the main sequence where it will spend most of its life. The temperature fluctuations in the star as it gets older can be seen as it travels up towards the giant branch, which is the near-horizontal line at the top of the graph. The purple circle marks where profile669.data is in the star's life, right as the star begins to expand dramatically near the end of its life. The star dies at about 9.5 Myr, with a radius exceeding 100 times that of the sun.

3.2 SPH code and *StarSmasher*

StarSmasher, the smoothed particle hydrodynamics (SPH) code developed by Dr. James C. Lombardi Jr. and collaborators, is where all the magic of this project takes place. *StarSmasher* is the code that allows us to create models of stars and put them into realistic dynamical collisions. The code uses a Lagrangian method to approximate a continuous fluid as discrete particles, each carrying physical variables such as mass, density, position, velocity, internal energy, pressure, and temperature, for example [29]. The process of this approximation involves the use of smoothing kernels, which can be thought of as a pocket of the continuous fluid [1]. Within the

pocket of fluid, only the effects of the surrounding particles inside the kernel are considered on the main particle. These particles are then referred to as the neighbors of the main particle and contribute to the neighbor number parameter, if they are within the kernel radius or about twice of the smoothing length.

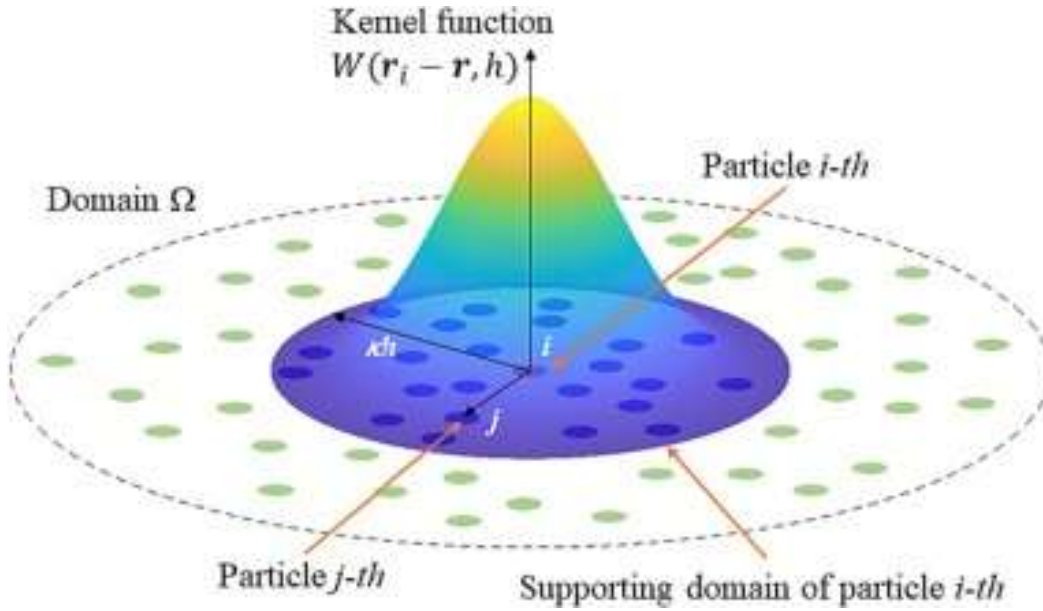


Figure 18: A diagram of the smoothing kernel of a particle i [30]. Particles inside of the kernel function radius can have physical effects on the particle of interest i while outside particles in the domain do not.

The smoothing length of the particle i depends on the number of particles that the user inputs. More information about the kernel function is covered in Backflip et al. 2025 [31].

3.3 *StarSmasher* Relaxations

Before a collision can be made in *StarSmasher*, we first need to take the profile file from MESA and move it to a machine with *StarSmasher* installed so we can use its data to create an SPH model of the star. Using MESA data, *StarSmasher* constructs a model of the star out of SPH particles placed onto a symmetrical lattice structure. Once the run is set to start, the particles are free to interact with one another via gravity and pressure forces. Ideally, the particles end up achieving hydrostatic equilibrium at the same radius value that was in the MESA profile; this is a process called relaxation. It's important to take the necessary steps to make sure that the final model from the relaxation is as realistic as possible, and to do this, there are a few parameters to keep your eye on.

One of the important parameters in the input file, `sph.input`, is *equalmass*, which controls the distribution of particles within the star model based on density values. In the code, the mass of each particle is proportional to density in the form $m \propto \rho^{1-\text{equalmass}}$, so an *equalmass* equal to one is associated with the mass of each particle comprising the star being equal, so you'll see a higher number density at the core than at the outer layers. Conversely, an *equalmass* value of zero is associated with an equal number density of particles throughout the star, thus particles down in denser regions must be assigned higher mass values than those in the outer layers to accommodate. Since *equalmass* is associated with the particle number density throughout the whole star, values closer to one will create a star with far more particles down towards the core and it becomes more computationally expensive for the code to resolve the motion for each individual particle. Because of this difficult resolution, the code is forced to take smaller time steps and the amount of time for a run to complete is subsequently increased. The opposite effect occurs when *equalmass* is set to values closer to zero, and subsequently the code can run much faster with much larger time steps. The disadvantage with a lower value for *equalmass*, however, is that the entire core of the star can sometimes be approximated as just one SPH particle (a core point) which cannot be plugged back into MESA to observe its evolution. So, while a lower value allows runs to be completed more efficiently, it does have some downsides.

Another important parameter in the `sph.input` file is *trelax*, which controls the time scale for an artificial drag force. In *StarSmasher* relaxations, the star model will initially oscillate as it tries to reach hydrostatic equilibrium. Without a drag force in play, some of the particles of the star can get blown away with enough kinetic energy to reach escape velocity and never return. To avoid this, *trelax* implements an artificial drag force on each particle so some of their initial kinetic energy is dissipated. The relationship between the artificial drag force and *trelax* is

$$\overrightarrow{F_{drag}} \propto -\frac{\vec{v}}{trelax}. \quad (3.3.1)$$

It's worth noting that $\overrightarrow{F_{drag}}$ is proportional to minus the velocity of the particles, signifying that the drag force's purpose is to counteract the motion of the particles, and what value we choose for *trelax* is in response to the initial velocity of the particles. This means that there is a considerable amount of trial and error in the process of choosing a value for *trelax*. Another

parameter, *treloff*, marks the time when the artificial drag force is turned off so that the stellar relaxation can return to a dynamical calculation and result in a realistic star model.

After a star relaxation in *StarSmasher* is complete, we can check if it was a good one to keep from a couple different graphs that the code generates, each one telling us different information about the star. The first one I will show here is a suite of six separate parameters versus the radius for my best relaxation of the $24 M_{\odot}$ star, made from 20,000 SPH particles.

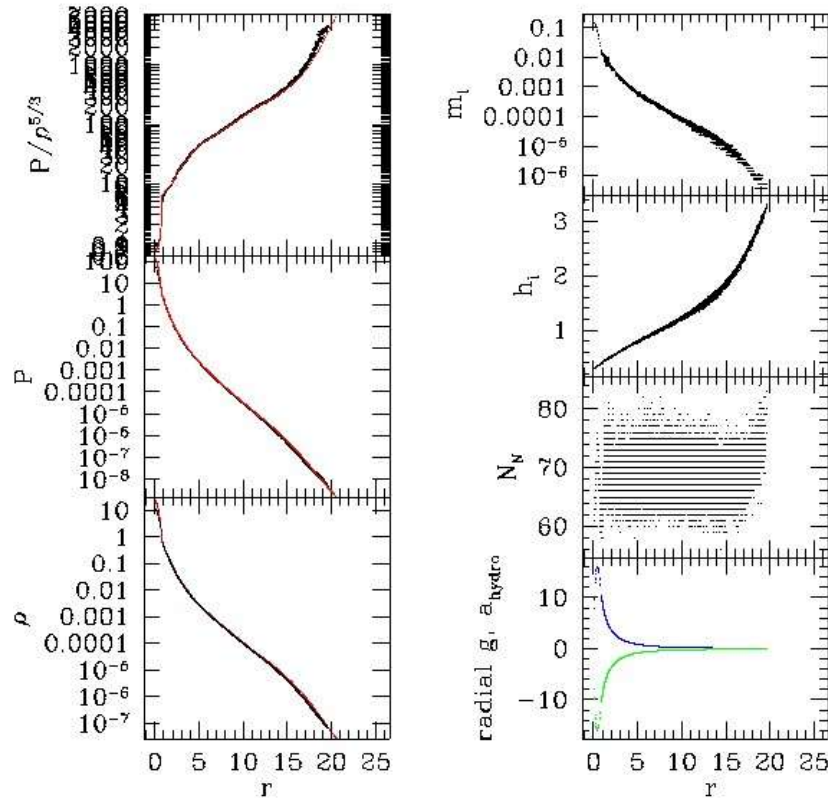


Figure 19: Seven different graphs from the col0060.sph file produced in the $24 M_{\odot}$ star relaxation run. The top left plot is the ratio of pressure over density to the $5/3^{\text{rd}}$ power, the middle left is pressure, and the bottom left is density, all versus radius from the center. The plot on the top right side is particle mass, the next one down is the smoothing length h , The next one down is the neighbor number, and the one on the bottom is radial gravitational acceleration and hydrodynamic acceleration, all of which are versus radius. It's worth noting that in *StarSmasher*, the units for G , $M_{\odot}$, and $R_{\odot}$ are all equal to one, thus density is in units $\frac{M_{\odot}}{R_{\odot}^3} [1]$.

The three graphs on the left column deal with the density ρ and pressure P of the star versus its radius, with the top graph showing the ratio of the pressure over the $5/3^{\text{rd}}$ power of density versus

radius. The black dots plotted on these graphs represent the individual SPH particles that comprise the star, and looking closely at the graphs reveals a red line underneath the black dots. This red line is the calculated expectation from MESA for the associated parameter in the star. A good relaxation will show the black dots lining up closely with the red expectation line, showing that the particles are behaving how they should, and the relaxation produced a realistic model of the star.

As for the four graphs on the right column of the figure, the top one is particle mass versus radius; *equalmass* for this relaxation was equal to 0.4, so the particles are shown to lose mass the farther they are away from the center of the star. The second plot from the top of the figure is the smoothing length discussed in section 3.2. The third plot from the top of the figure is neighbor number N_N also discussed in section 3.2. The final plot on the right column is the radial g (gravitational acceleration) and hydrodynamic acceleration versus the radius of the star, which should mirror each other in a good relaxation, demonstrating hydrostatic equilibrium [1].

The next plot that is important for judging relaxation runs is contained in the energy0.sph file generated by the code. This file contains information regarding the total energy of the system, the kinetic and potential energy, and the internal energy of the system.

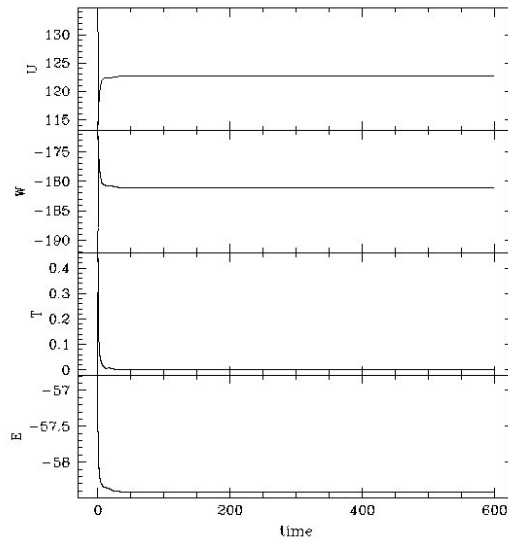


Figure 20: The plot from energy0.sph for the $24 M_{\odot}$ star relaxation run. From the top of the figure to the bottom, the order is internal energy, potential energy, kinetic energy, and total energy of the star during its relaxation. Although it is difficult to spot in this graph, the plot for kinetic energy usually shows a fluctuation or spiking around where the artificial drag force has been turned off. After this point, ideally, the plots for each of these values should level out to a constant value showing that the star has become

static. It's worth noting here that the units for energy are $E = \frac{GM_{\odot}^2}{R_{\odot}}$ and the units for time are $t = \sqrt{\frac{R_{\odot}^3}{GM_{\odot}}}$, values for which are contained in appendix A [1].

3.4 Dynamical Collisions

Once a good star relaxation has been completed, it's time to throw it at a BH in a dynamical collision, the main focus of this research. In such a simulation, the final out.sph file is moved to a new directory with an input file and other files associated with a collision simulation. Once the run is set to start, the star model from the out file is placed in space with a BH that is treated as a point mass and gravity does the work from there.

How exactly the collision takes place depends on the input values for parameters such as periapsis separation, apoapsis separation, and BH mass. Depending on these values, the code can set up the initial conditions in such a manner to produce a TDE or PTDE between the star and BH that is accurate to reality. The way that *StarSmasher* produces output files in a dynamical collision simulation is the same way that it does in a relaxation, with one output file being produced every time step and with each being a snapshot of the system at that point in time. The rate at which these files are produced is dependent on the resolution of the particles of the star. In a more direct, violent interaction with a small periapsis value, the resolution of the particles' motion is more difficult for the code to calculate and, therefore, the rate at which output files are produced is much slower. This turns an already long period of time for a run to complete (weeks or even months) into a potentially longer period. Interactions where it is not as computationally expensive to resolve the motion of the star's particles like PTDE's result in a shorter amount of time to complete a run.

4 Preliminary Results

Over the course of the fall semester, I have made progress in this research project by first re-learning the relevant code, MESA and *StarSmasher*, from my junior year independent study, which mainly served as a way to learn and familiarize myself with the tools that would be used in this project. In the process of familiarizing myself with the relevant code, I have made a satisfactory model of the $24 M_{\odot}$ star after communicating with the authors of the previously mentioned paper, Kyle Kremer and Fulya K ro lu [7]. Some of the important values that are

relevant to my research from their CMC data include the average age of giant stars when they undergo tidal disruption, 7 Myr, the average mass of the star and black hole, $24 M_{\odot}$ and $30 M_{\odot}$ respectively, the average radius of giant stars in the interactions, about $20 R_{\odot}$, and the fact that 95% of the giant stars were core helium burning at their age. It's worth noting that a majority of the giant star-BH interactions took place early in the evolution of the cluster, which is when there was a higher population of giant, massive stars. I have also made various collision simulations with this star model, of which I plan to show the results of my best one in this section.

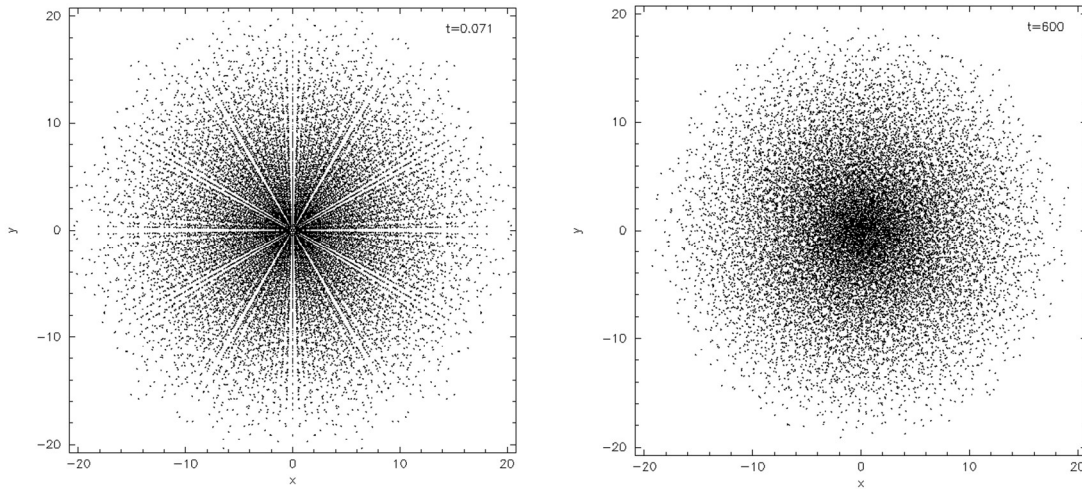


Figure 21: The first (left) and last (right) output files from my best $24 M_{\odot}$ star relaxation in *StarSmasher*. The first output file clearly shows the lattice structure that the particles are placed on initially while the last output file shows the final states of the particles where they have situated themselves to balance internal pressure and gravity.

It's worth noting that the relaxation run that produced this model had an *equalmass* = 0.4 and an *NNOPT* = 50. With a successful relaxation completed, I copied the final output file from the relaxation run into my collision directory where I would set the star model to collide with a BH of $30 M_{\odot}$. The following figure shows the collision in our visualization software, SPLASH [32].

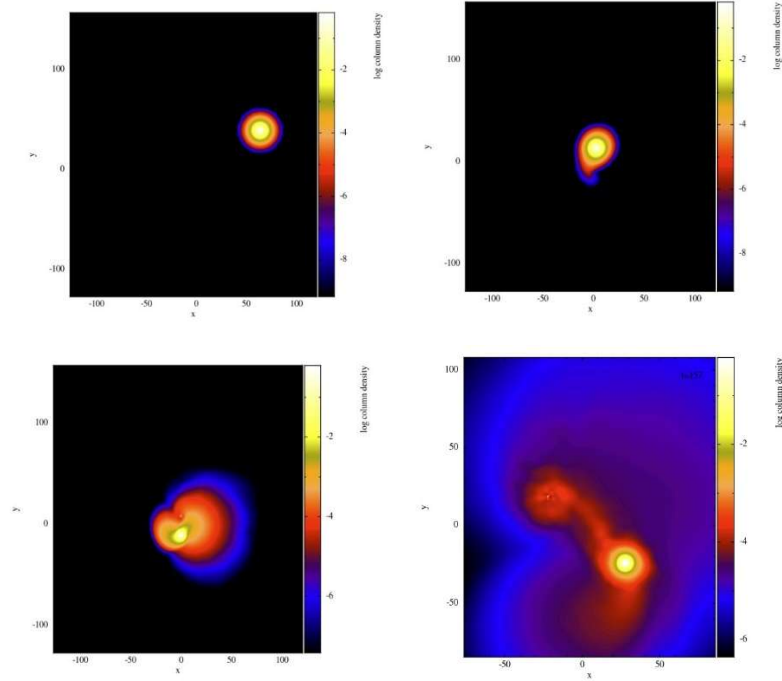


Figure 22: Four images of various output files from my collision simulation between a $24 M_{\odot}$ star and a $30 M_{\odot}$ BH, they are ordered chronologically from the top left to the bottom right as the star approaches the BH, reaches its first pericenter passage, and undergoes tidal disruption. The type of interaction pictured here is a PTDE where the star gets its outer layers stripped away by the BH while the inside of the star survives the interaction. In this particular case, the star remnant is still gravitationally bound to the BH so it will return eventually for another pericenter passage.

The way that SPLASH displays the interaction in figure 21 is with the log of the column density in $\frac{g}{cm^3}$. The brighter, yellower colors near the top of the axis represent higher number density of SPH particles while the darker, bluer colors near the bottom of the axis represent lower number density values. It's also worth noting that the x and y axes are in $R_{\odot}$. The associated energy graphs for this collision are presented in the following figure.

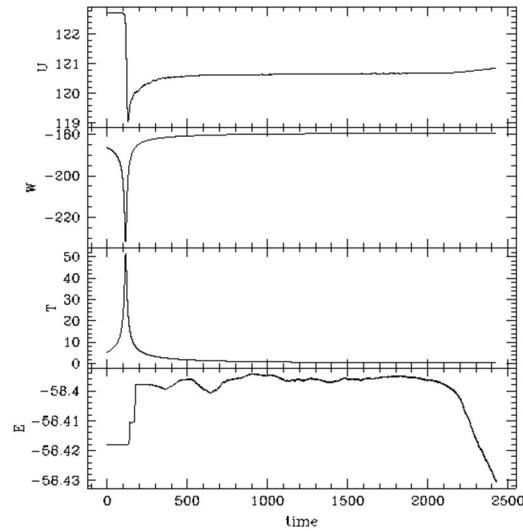


Figure 23: The energy plots for the collision presented in figure 21. The spike in kinetic energy and mirrored dip in gravitational potential energy is where a collision occurred between the star and BH. For the time being, this is the only collision that has happened between the two objects, but another could happen as we will see from the next figure.

The next graph I will show is data from the massAndMore.out file, which is read by the Python code Hypermongo, developed by Allegheny College alumni Jacky Tran 22'. Hypermongo can take many different parameters of the collision from massAndMore.out and construct a plot to analyze how they affect each other or change throughout the collision.

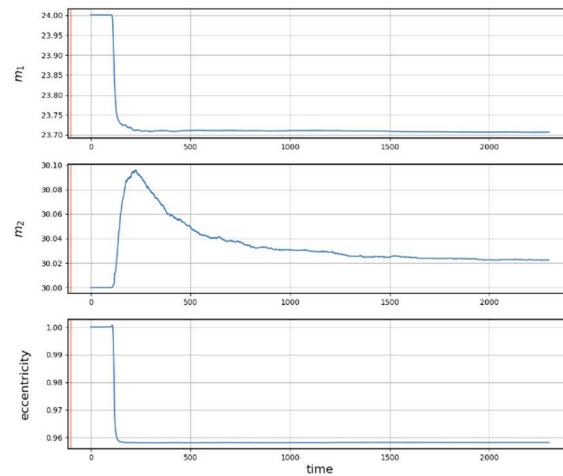


Figure 24: The massAndMore.out file associated with the collision from figure 21. I chose to plot the mass of the star m_1 , the mass of the BH m_2 , and the eccentricity of the star's orbit around the BH. Something noteworthy here is how, after the collision takes place, the orbit's eccentricity decreases to a value slightly less than one. From table 2.2.1, this tells us that the star will eventually come back for another pericenter passage to get tidally disrupted by the BH again, which can be calculated from Kepler III.

5 Simulation Results and Analysis

Over the course of this project, I have run over 20 useful simulations with the supergiant star and stellar mass black hole obtained from Fulya Kiroğlu and Kyle Kremer’s CMC data mentioned in section 1.2. Many of these simulations were of PTDE’s where I analyze how a single pericenter passage with the black hole affects the star remnant, both its physical properties as well as the properties of its resulting orbital path. However, I allowed some of these collisions to run up until total tidal disruption in order to illustrate the effects of tides on the star remnant, a phenomenon discussed in Kremer et al 2022 and Press & Teukolsky 1977 [25] [26].

My original game plan for evolving stellar remnants for this project was to situate a “perfect” collision which would result in a high amount of mass loss and ultimately ejection of the remnant from the system. My reasoning for a desired condition such as this has to do with the need for my simulations to adhere to reality. In order to justify evolving my collision products in MESA, they need to undergo the disruption process in such a way that results in one of two outcomes: either the orbital period of the remnant becomes exceedingly long after a passage or the remnant gains enough kinetic energy to escape the black hole’s clutches all together. The reason that these are the two desired outcomes is because they both allow the remnant enough time to contract and resume stellar evolution post-pericenter passage.

The problem with my original line of thinking has to do with the orbital energy that contributes to the tides of the star, discussed in section 2.6. The presence of E_{tides} in equation (2.6.1) will decrease the overall value of orbital energy and result in a shrinking semimajor axis and orbital period, slowly dragging the star closer and closer until it is completely disrupted by the black hole. As discussed in section 2.6, the orbital period of the star is proportional to the periapsis distance to the ninth power,

$$P \propto a^{\frac{3}{2}} \propto r_p^9. \quad (5.1)$$

So at large periapsis values, the orbital period increases exponentially to very long times than it otherwise would have at small periapsis values. The problem here is that the desired large orbital periods are locked to large periapsis values, where only small amounts of mass get stripped from the star. If I were to stick to only considering single star-BH interactions, then I would lack the

truly intriguing cases of remnants with much larger mass loss percentages in MESA evolutions. And while stellar remnant ejection from a single star-BH system is still possible, as we can see from E_{kick} in equation (2.6.1), the ratio of my star and black hole mass is too close to unity for a large enough amount of matter to get stripped from the star (which is what is necessary for E_{kick} to exceed the sum of E_{bind} and E_{tides}).

With this problem in mind, I shifted my gaze to scenarios beyond the interactions that I had been conducting previously. Instead, I could apply results from my simulations to other scenarios that could result in ejection of the remnant. The first, seemingly obvious, approach was to simply increase the mass of my BH by a large factor. The benefit of this approach is that I can continue making single star-BH simulations with a much higher likelihood of the collision remnant receiving a large kick velocity. Another scenario I considered is that of a star encountering a binary black hole system (or a BBH). Systems such as these are the subject of Emma Chambers’ ’24 and Hans Vanderzyden’s ’25 respective research projects, where they analyze the effects of tidal disruption on the BBH after a star collision. The outcome of three-body systems such as these is unpredictable, with one such possibility being the ejection of a stellar remnant that could have been partially disrupted after many close passages to the black holes.

5.1 Resulting Orbits of Supergiant Star – Stellar BH Interactions

Before I consider the evolution of collision products, I want to discuss the resulting orbital mechanics of my single star-BH interactions and how they tie into results from Press & Teukolsky 1977 [26]. The data showcased in this section is over the span of 12 different simulations I have run with the $30 M_{\odot}$ BH and the $24 M_{\odot}$ supergiant, each with varying periapsis values.

Run Number	Periapsis ($R_{\odot}$)	Orb. Period (years)	Mass Loss (%)
1	5	0.0027	14.10
2	10	0.058	3.20
3	15	0.36	1.20
4	20	1.75	0.46
5	25	8.43	0.2
6	30	40.97	0.07
7	35	221.17	0.018
8	40	1247.22	0.0046
9	45	9513.20	0.0005
10	50	66148.02	0.000025
11	100	151988965.84	0
12	200	9.54E+10	0

Table 2: The periapsis, orbital period, and percentage mass loss values of ten of my single star-BH interaction simulations.. Both orbital period and mass loss were calculated from the data from the massAndMore file read by Hypermongo.

For the first ten runs, I increased the periapsis value by $5 R_{\odot}$, ending with a value ten times that of the initial value and roughly 1.8 times that of the black hole's tidal radius. As can be seen from the data, orbital period increases exponentially as the periapsis distance increases and the opposite is true for the amount of matter stripped from the star. At a periapsis distance of $5 R_{\odot}$, nearly 14% or $3.4 M_{\odot}$ is ripped away from the star and this value decreases exponentially with a value of 0.000025% or $6 \times 10^{-6} M_{\odot}$ at a periapsis distance of $50 R_{\odot}$. This relationship is even more apparent at larger periapsis values; at distances such as 100 or $200 R_{\odot}$, the orbital period blows up to and even exceeds billions of years. The relationship between these three variables is visualized as follows:

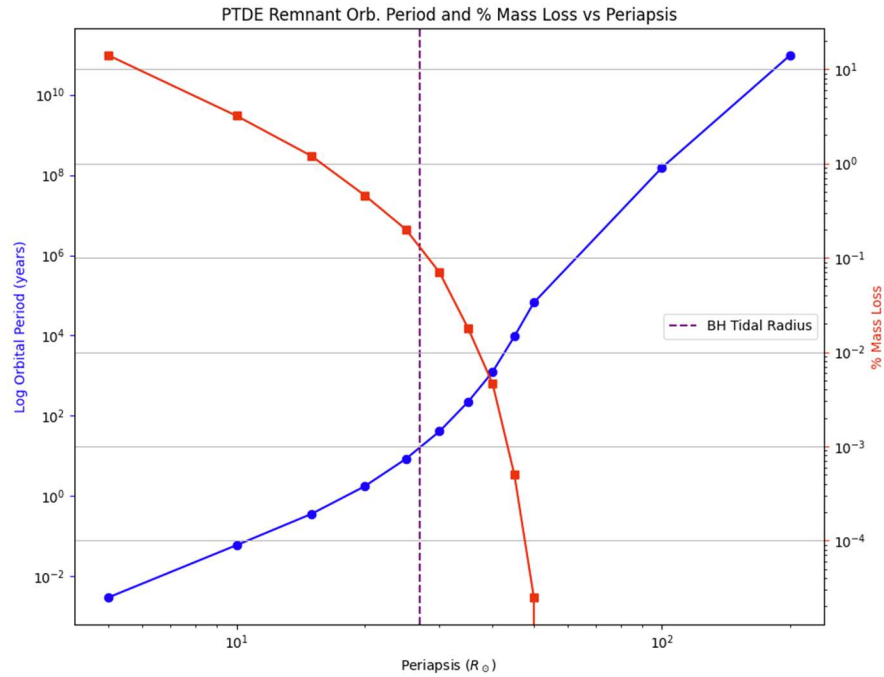


Figure 25: A plot of orbital period (blue curve) and percentage mass loss (red curve) as a function of periapsis distance on a log-log-log scale. Each circle on the orbital period curve that lines up vertically with a square on the mass loss curve corresponds to one of the runs completed in table 2. It's also worth noting that the vertical dashed purple line is the tidal radius of my BH ($27 R_{\odot}$). On this scale, the inverse relationship between orbital period and mass loss as periapsis separation increases is more apparent. It's also worth noting large orbital period values do not correspond to values on the mass loss curve because there was no mass transfer in those cases.

As expected, orbital period drastically increases as we consider distances farther away from the black hole, qualitatively lining up with the results from Press & Teukolsky [26]. As discussed in section 2.6 and the beginning of chapter 5, plotting orbital period and periapsis on a log-log scale should show a slope approaching nine at large values. How does my data hold up with this result?

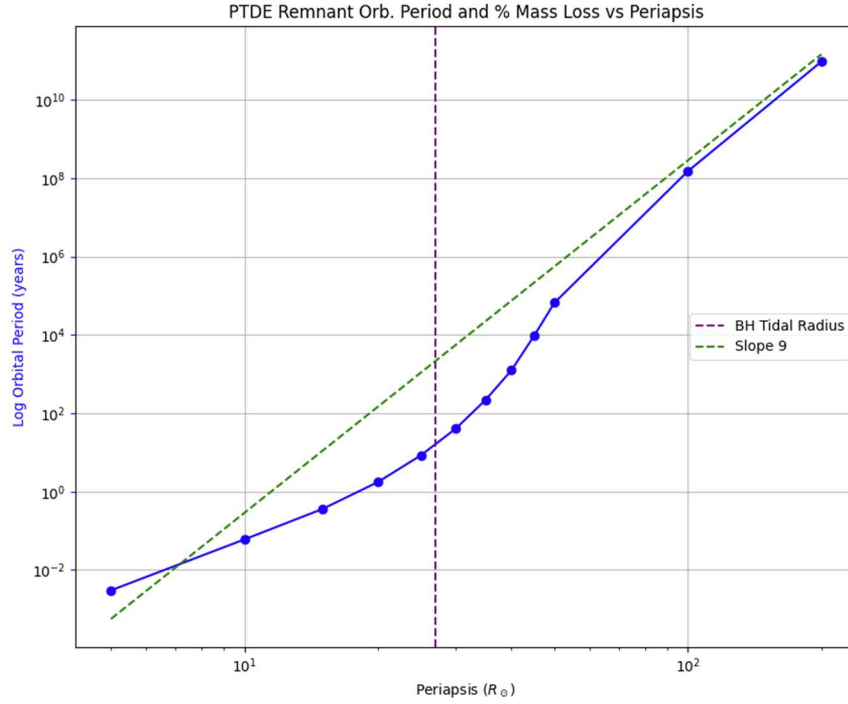


Figure 26: Figure 24 with the addition of a dashed green line with a slope of nine, representing the results from Press & Teukolsky 1977 [26].

As can be seen from Figure 25, my data lines up quite well with the mathematical results from equation (2.6.3). The slight change in concavity from smaller to larger periapsis values has to do with the weighting of different terms associated with the oscillation modes of the star in equation (2.6.3). Periapsis values closer to the radius of the star will cause later terms in the summation to contribute more to the overall value of E_{tides} , resulting in slopes greater than nine. This effect fades as the ratio of R_*/R_p approaches zero, resulting in a larger contribution from the first term. Two of the orbits used as data for figures 24 and 25 are pictured below.

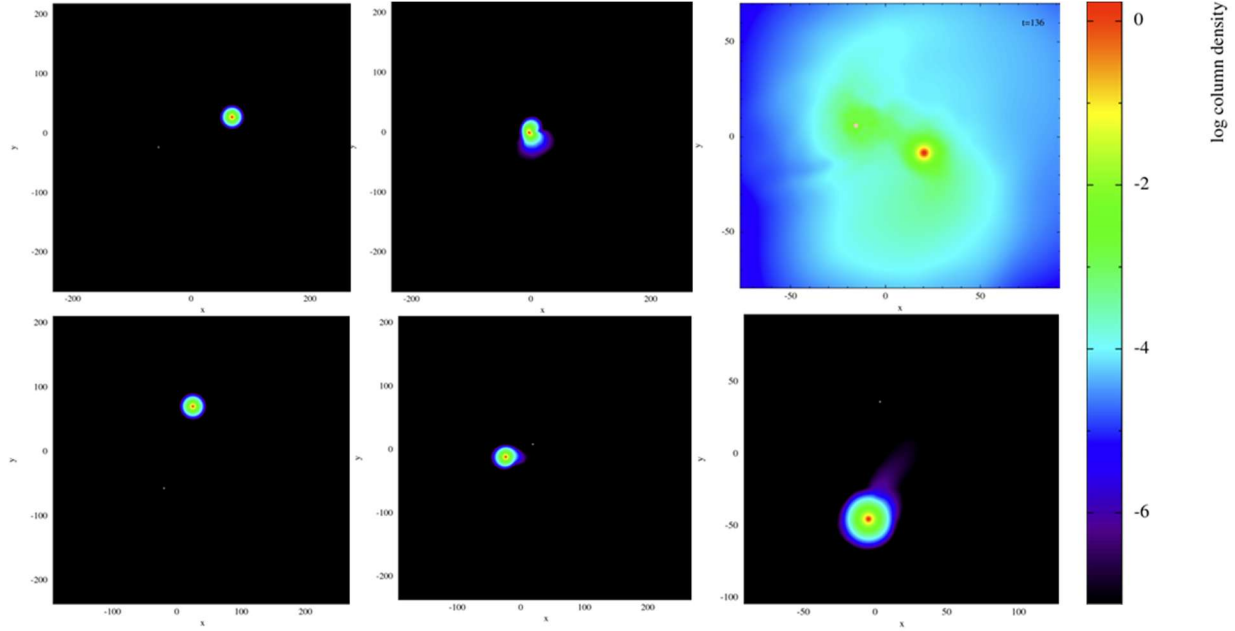


Figure 27: Two different simulations taken from the data showcased in table 2 and figures 24 and 25, (top three frames from the $r_p = 5 R_\odot$ case and bottom three frames from the $r_p = 45 R_\odot$ case). Each set of three frames spans one pericenter passage. The $r_p = 5 R_\odot$ simulation resulted in high mass loss from the star and a small orbital period, resulting in the star and BH becoming locked in a close dance surrounded by diffuse stripped gases. The $r_p = 45 R_\odot$ simulation resulted in much less mass loss and a comparably large orbital period, the second and third frames showing just a slight tidal bulging of the star as it continues along its new orbital path. The star remains bound to the BH in both cases since they are of relatively similar mass. Videos of the collision can be played via this link:

<https://www.youtube.com/watch?v=6KoojP7LE10> (top) and this link:

<https://www.youtube.com/watch?v=6tL7d4Ubc8A> (bottom).

In section 2.6, I mentioned how the size of the star's orbit would shrink over time due to orbital energy being lost to tidal oscillations. This effect is more easily seen over the course of many pericenter passages. The following figure showcases this effect throughout the course of one simulation that I had run over a long period of time, long enough for multiple pericenter passages to take place.

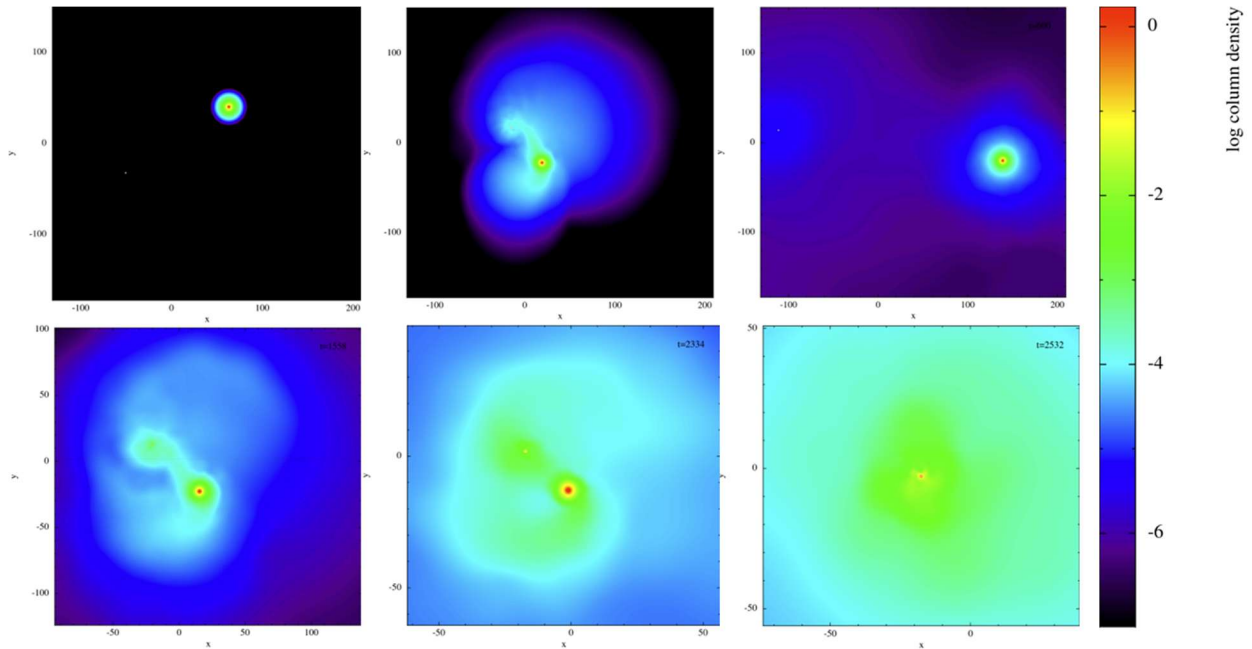


Figure 28: Six frames from a single simulation of the supergiant star orbiting the stellar mass BH, taken all the way until total disruption. The first frame showcases the star on a parabolic trajectory toward the black hole (represented by the small white dot). This is followed up by images of the first pericenter passage at about $r_p = 0.3r_t$, apoapsis of the first orbit, and two other pericenter passages before complete disruption of the star. The last two frames are zoomed in so we can more easily see the smaller orbital path of the remnant at later times. The star went through twelve total passages before it was ripped apart by the black hole, over the span of about one and a half months. A video of this collision can be viewed via this link: <https://www.youtube.com/watch?v=VTB5tz3cARg>

5.2 Remnant Evolution Post Stellar Mass BH PTDE's

A vast majority of the simulations I have run over the course of this project have been with a stellar mass black hole that is comparable in mass to the supergiant star ($30 M_\odot$ BH and $24 M_\odot$ star). The upside of choosing mass ratios like this for collisions is that they represent collisions that are more likely to happen inside a cluster. Depending on the age of the star cluster you consider, stellar mass black holes are much more common than their more massive siblings, especially after most of the more massive giant stars have died out in supernovae, leaving behind said stellar mass black holes throughout the cluster. So, the collisions I have made with this black hole model represent TDE's and PTDE's that are more likely to occur in the hypothetical globular cluster I am considering.

5.2.1 Low Mass Loss Case

The first interaction I will consider in this section is a case with a large periapsis distance between the star and the black hole, $45 R_{\odot}$. Large periapsis distances like these will result in an admittedly small amount of mass loss experienced by the star but with a comparably large orbital period, allowing the remnant plenty of time to contract and continue evolving. The first pericenter passage of this interaction is shown below.

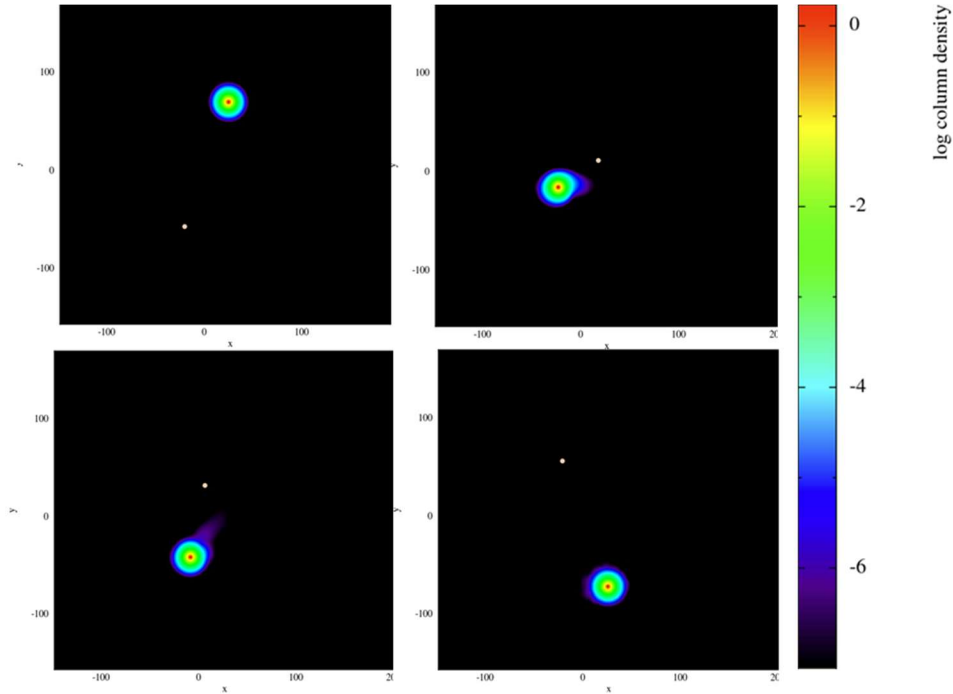


Figure 29: Four important frames from the $r_p = 1.67r_t$ case. The star follows a parabolic trajectory toward pericenter, where it then gets tidally stretched by the black hole without a large amount of mass getting stripped off. The resulting tail can be seen forming in the top right frame and falling back onto the star in the bottom two frames.

As can be seen from table 2 and the figures from section 5.1, the mass loss from this case was just about 1% of a solar mass, making the resulting remnant basically the same star as the run began with. However, this periapsis distance was one of the smallest that allowed a long enough orbital period to see stellar evolution post-pericenter passage, making this run a prime candidate for a realistic case of stellar evolution post-PTDE of a high-mass star around a stellar mass black hole. The next thing we can do with simulations such as these is importing data from the collision to a directory containing MESA to view the resulting evolution of the collision product. This evolutionary track on the HR diagram is shown below.

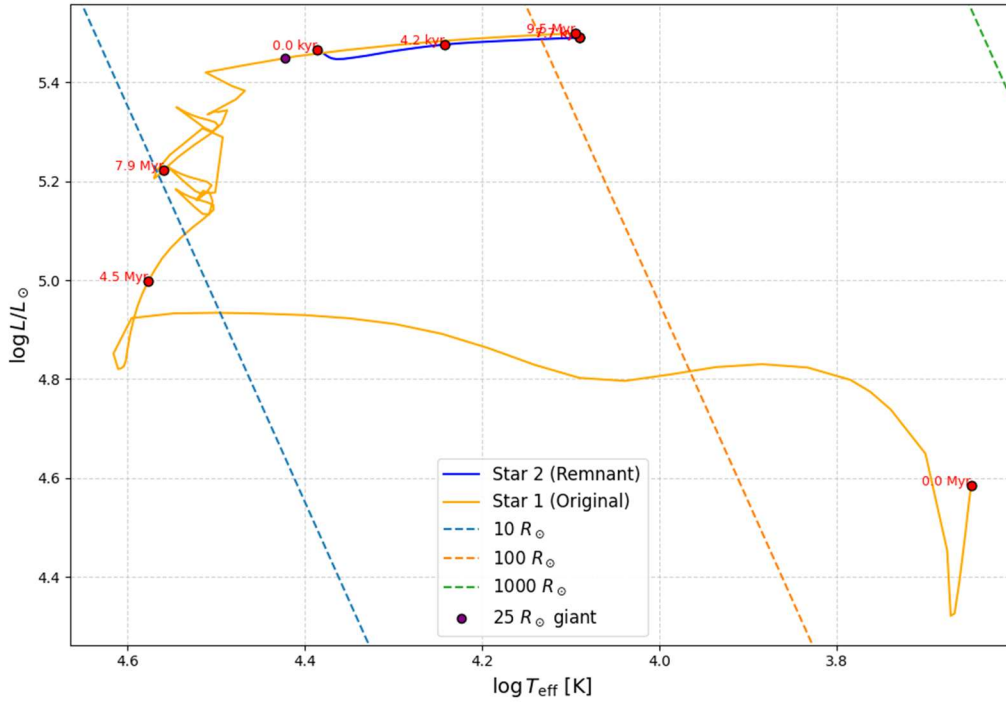


Figure 30: The evolutionary track of the collision product (blue line) shown next to the evolutionary track of the original star (orange line) on the Hertzsprung-Russell Diagram. We can see the remnant starts off at a similar luminosity and surface temperature as the original star as it matches up with the original star's HR track through later times. We can also see the remnant continuously cooling down with a slight rise in luminosity, signifying that it is expanding. Important timestamps throughout the star's life are marked in red on both HR tracks, showing that the remnant dies about 7.7 kyr (7.7×10^3 years) after the collision, a span of time less than the remnant's orbital period of about 9 kyr.

How does the composition of this collision product differ from the star had it never undergone tidal disruption? The profile files generated by the MESA run contain information about the composition of the star as a function of radius. If we take a look at the first profile file generated in the MESA run of the remnant (corresponding to the shortest amount of time post-collision) and compare it to a star of the same mass and radius that had never undergone tidal disruption, then we can get an idea of how the remnant would look composition-wise compared to a star of the same mass and radius. This is shown in the following figure:

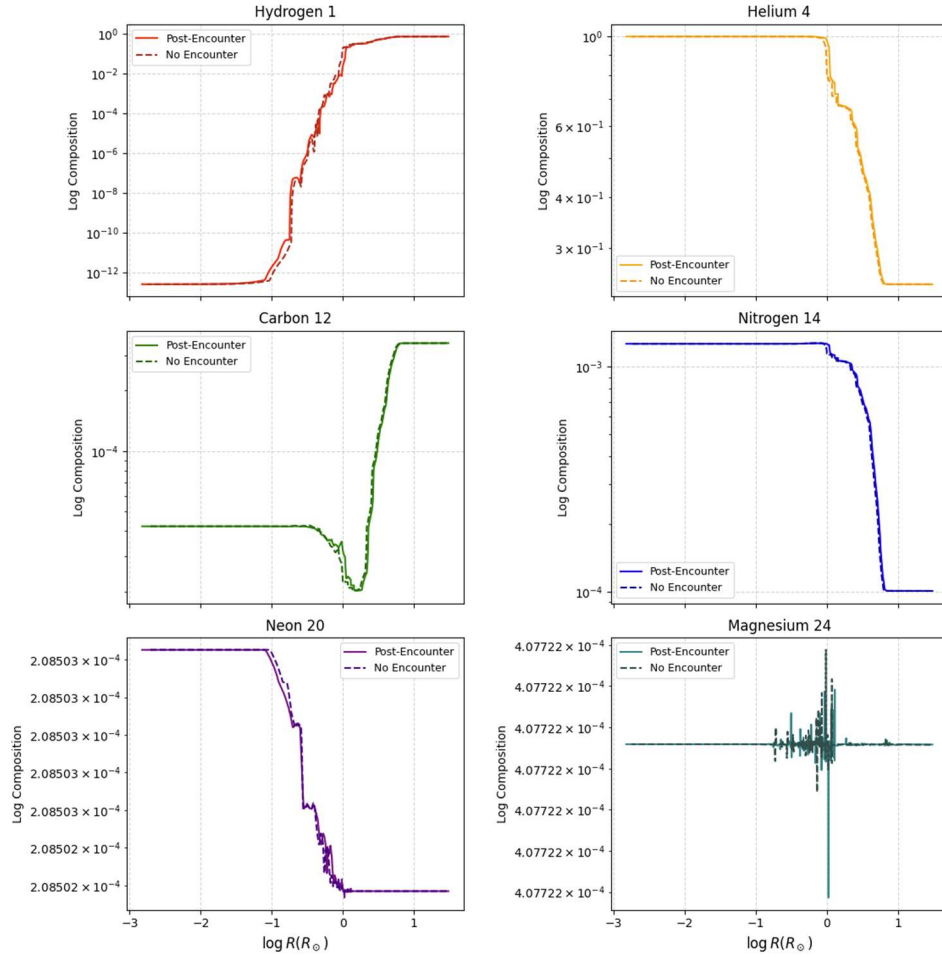


Figure 31: Composition curves for the elements Hydrogen 1, Helium 4, Carbon 12, Nitrogen 14, Neon 20, and Magnesium 24 for the remnant post-collision (solid line) and a star of the same mass and radius that has never had any mass loss (dashed line). Since we are working with an early supergiant star here, we can see a large concentration of Helium 4 in the center of the star that is gradually replaced by Hydrogen 1 as we travel toward the outer layers. The amounts of Carbon 12, Nitrogen 14, Neon 20, and Magnesium 24 are comparably much smaller than Hydrogen and Helium. Since the remnant only experienced a small amount of mass loss, the solid curves match up with the dashed curves quite well for all elements all the way out to the surface. This means that observations of a remnant like this case would not differ much from observations of a similar star that had never undergone tidal disruption.

5.2.2 Higher Mass Loss Case

The next case I will look at in this section is a more exciting case with a higher amount of mass taken from our poor supergiant star. In this run, I allowed the simulation to carry on past one pericenter passage and I instead took the data from after two pericenter passages around my $30 M_{\odot}$ stellar mass BH, resulting in nearly 30% of the star's mass getting stripped away. It is important to note that this star was still bound to the black hole after the second pericenter passage and had the simulation been allowed to run over a longer period of time, we would have

seen the star slowly fall into the black hole with a shrinking semimajor axis and periapsis distance with each subsequent pericenter passage until the star was completely disrupted, not unlike figure 27.

This effect is the result of orbital energy lost to tides, topics discussed in sections 2.6 and the beginning of chapter 5. This is how this particular PTDE would play out in reality, so in order to justify evolving the remnant in MESA, we can apply a hypothetical scenario to this situation that would result in our stellar remnant either gaining an extremely long orbital period or becoming unbound from the system altogether. This ties into the discussion in the beginning of this chapter about three-body systems like stars encountering BBH's in star clusters. The inherent instability of systems like these brings the possibility of an unbound remnant with a high mass loss percentage back onto the table, at least when it comes to massive stars encountering stellar mass black holes. So had this remnant been the result of our titular supergiant protagonist encountering a binary of two stellar mass black holes of roughly similar mass, we can move forward with evolving it in MESA under the assumption that this type of remnant is a real possibility in this scenario. The simulation that produced this remnant is pictured below:

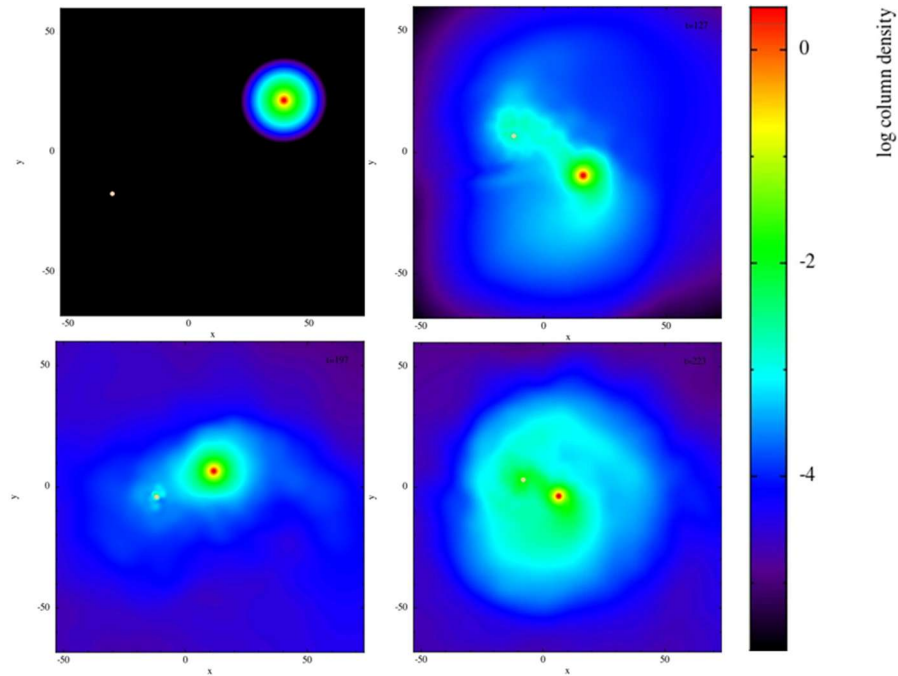


Figure 32: Four frames from the simulation that produced the $17 M_{\odot}$ remnant mentioned above. The first frame shows the star on a parabolic trajectory toward the black hole. The second frame shows the aftermath of the first pericenter passage ($r_p = 0.19r_t$). The final two frames show the star moving

towards the second pericenter passage and the aftermath of said passage. The log column density gradient has been altered to better show the denser tidal tails formed in this collision.

Had the remnant produced by this collision been in a situation that resulted in a long orbital period or stellar ejection, how would it have continued to evolve? Plugging data from the collision product into MESA and initiating a run resulted in the following information:

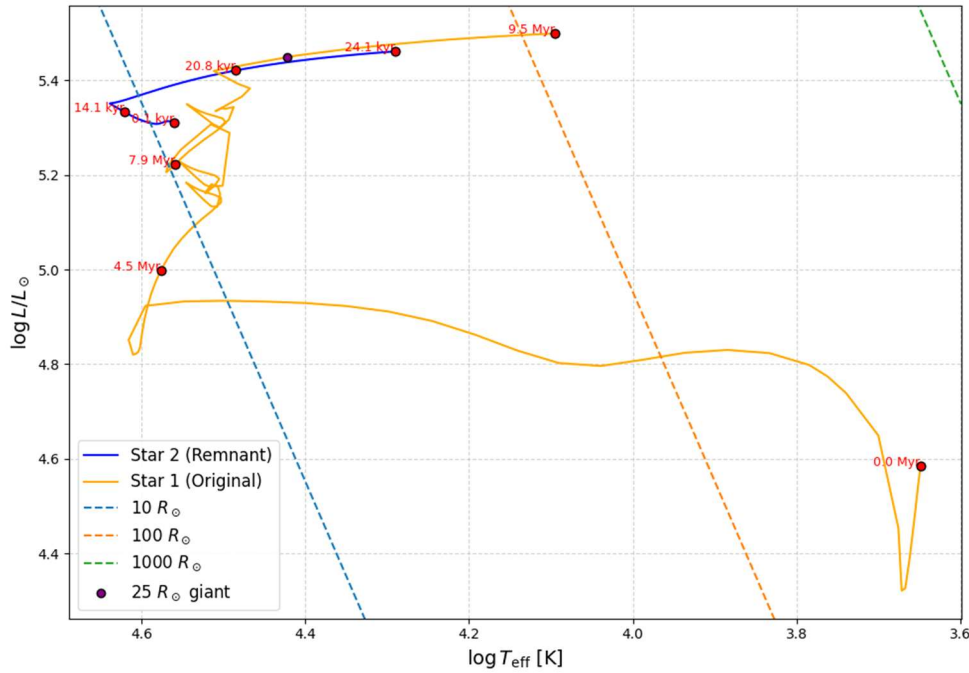


Figure 33: The evolutionary path of the $17 M_{\odot}$ remnant resulting from the collision showed in figure 30. Due to the large amount of mass stripped from the star post-collision, it is considerably smaller and at a slightly lower luminosity than it was originally, despite having a higher surface temperature. The HR track of the remnant indicates that it begins to contract, heating up and increasing in luminosity before it starts moving toward the giant branch and expanding to larger radii. The star continues to expand, indicated by the slight increase in luminosity and drastic decrease in temperature as it moves along the giant branch (matching up closely with the path of the original star). The remnant's life ends about 24 kyr after the collision took place, a span of time that this remnant would not have seen unless given a large kick velocity.

How does the composition of this remnant look compared to the composition of a non-collision product star of the same mass? Once again, the profile files from the MESA run of this remnant can give us this information. Because of the higher amount of mass loss that produced this remnant, we can predict that its composition profile will show a larger discrepancy than the $r_p = 1.67r_t$ case showed earlier in this section.

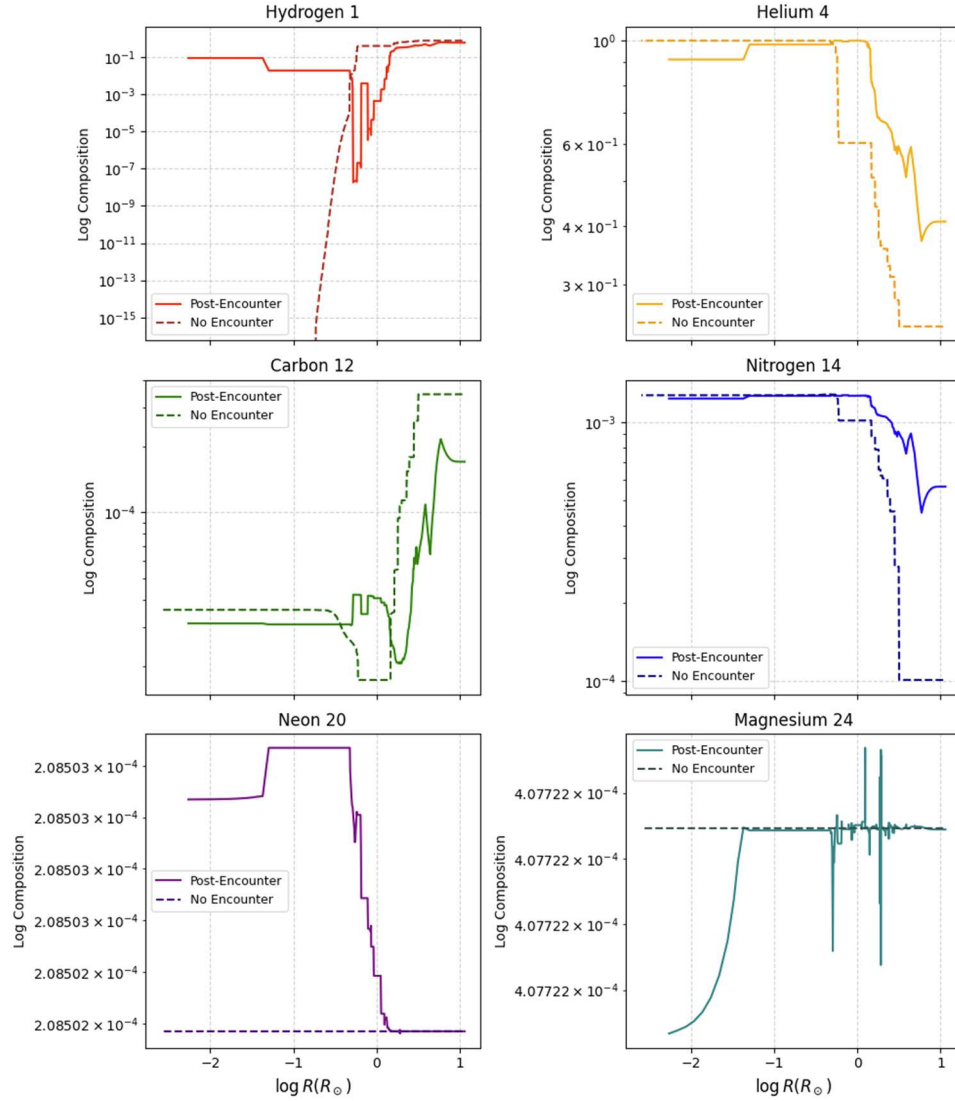


Figure 34: The composition curves for Hydrogen 1, Helium 4, Carbon 12, Nitrogen 14, Neon 20, and Magnesium 24 for the 17 M_⊙ remnant produced in the above collision. Because of the higher amount of mass stripping from the remnant, we see a larger discrepancy in composition at the surface compared to the normal star of equal mass. This shows that tidal disruption of the supergiant star results in a star with higher compositions of Helium 4 and Nitrogen 14 and slightly lower compositions of Hydrogen 1 and Carbon 12 at the surface. These anomalous surface compositions would be a giveaway that this star was the result of mass stripping and is potentially a PTDE product.

5.3 Remnant Evolution Post Massive BH PTDE's

As discussed in section 2.6 and the beginning of this chapter, PTDE's between a star and black hole of similar mass will result in a remnant with a relatively low amount of mass loss that will remain bound to the black hole for many subsequent pericenter passages due to orbital energy being imparted into tidal oscillations of the star. Because of this effect, it is difficult to initiate a run that will result in a large amount of mass loss from the supergiant *and* a long enough orbital period to see evolution in the star. To add on top of this problem, giant and supergiant stars tend to shrink in radius after tidal disruption if the mass of the core, $m_c > 0.214$ times the mass of the star, a result discussed in Hjellming and Webbink 1986 [33]. This is true of my blue supergiant, meaning that the star will shrink in size after each subsequent pericenter passage and the rate at which it loses mass per passage will decrease as well, making simulations with multiple passages take much longer.

However, the above effects become less of a problem as we increase the mass of the black hole in our SPH simulations. A phenomenon also discussed in section 2.6 was the kick velocity, closely related to E_{kick} in equation (2.6.1). The kick velocity is imparted into the remnant post-pericenter passage and is proportional to the amount of mass stripped from the remnant during tidal disruption. Large amounts of mass stripping will impart a higher kick velocity to the remnant, which can sometimes exceed the escape velocity of the black hole or even the escape velocity of the cluster that the star is a part of [2]. More massive black holes tend to result in higher mass loss from the star post-pericenter passage. So, if we want to increase the likelihood of a star remnant receiving a high enough kick velocity to drastically increase its orbital period, or even eject it from the system entirely, we need to increase the ratio of M_{BH}/M_* . However, it is important to point out that the rarity of these collisions occurring becomes greater as we increase the mass of the black hole, approaching IMBH and even SMBH territory. While we have sufficient evidence that SMBH's are usually located in galactic nuclei, IMBH's are still a subject of research, and we have little observational evidence of them besides their possible presence at the centers of globular clusters.

When determining the initial conditions for simulations with black holes of higher mass, we need to consider the properties of the star that we will have undergo tidal disruption. Equation 5

from Kiroğlu et al. 2023 can give us an idea of what r_p/r_T ratio we want to consider for a simulation to result in an unbound remnant [2],

$$\beta^{-1} = \frac{r_p}{r_T} = \zeta^{\frac{1}{3}} \left(\frac{\bar{\rho}_*}{\bar{\rho}(R)} \right)^{\frac{1}{3}}. \quad (5.3.1)$$

This equation shows the inverse of the critical penetration factor β (the numeric boundary between total disruption and partial disruption) as a linear relationship between the ratio of the average star density ρ_* over the density using the star's mass as a function of its radius $\rho(R)$ to the one third power and the coefficient ζ to the one third power [2].

If we fit β in the limit that $R \rightarrow 0$ for complete tidal disruption, then we can set $\rho(R) = \rho_c$ (the density of the core) [2]. ζ is determined by fitting the density ratio and penetration factor based on previous results from Kiroğlu et al. 2023, which ended up being approximately 5.6 for a main sequence star [2]. If we apply this value for ζ along with the values for average and core density from the MESA profile of our supergiant, then we can get a rough estimate for the critical penetration factor beyond which our star would be destroyed. This value ends up being $r_p/r_T \approx 0.045$. Since this is a rough estimate, we'll keep things safe by limiting our ratio as $0.05 \leq r_p/r_T \leq 0.1$ if we want to see a potentially unbound remnant. All the simulations I have run below $r_p/r_T = 0.05$ ended up in total disruption of the star anyway, hence why we're setting that as the lower boundary. This critical penetration factor differs greatly from the main sequence star collisions run in Kiroğlu et al. 2023, with a $r_p/r_T = 0.37$, on an order of magnitude greater than mine [2].

5.3.1 But Wait, There's (Not Quite) a Problem!

Before I move on to simulation results with higher mass BH's, it is important to note that not even one of my nearly 30 collision simulations resulted in an unbound remnant, if the run had an initial velocity of zero ($v_\infty = 0$). This is where my results mainly differ from those of Kiroğlu et al. 2023. In their paper, they found that PTDE's of main sequence stars around IMBH's often resulted in ejection of the remnant if it experienced sufficient mass loss [2]. However, remnants of my supergiant model that experienced similar mass loss percentages remained bound to the BH, shedding light on just how different PTDE's of giants and

supergiants are compared to main sequence stars. Some important results from collisions I have run are shown in the following table.

v_∞ (km/s)	r_p/r_T	M_{BH} ($M_\odot$)	M_{rem} ($M_\odot$)	Resulting e	Period (yrs)
0	0.032	24000	0	0.979	0.0149
0	0.07	24000	3.923	0.998	1.639
0	0.08	24000	10.022	0.996	0.789
0	0.1	24000	13.634	0.997	1.220
0	0.035	240000	0	0.9922	0.074
0	0.07	240000	5.254	0.9997	24.405
0	0.08	240000	10.007	0.9992	8.472
0	0.1	240000	13.516	0.9993	14.948
0	0.07	1000000	5.685	0.999957	541.550
40	0.08	240000	9.638	0.996	0.764
1000	0.08	24000	10.110	1.00072	N/A

Table 3: Eleven notable runs of the $24 M_\odot$ supergiant undergoing tidal disruption by BH's of varying mass, with mass ratios ranging from 1,000 to over 40,000 times that of the star. For runs with $v_\infty = 0$, the penetration factor seems to play the biggest role in determining the remnant mass while the resulting eccentricity seems to asymptotically approach 1 as BH mass increases by orders of magnitude. Any runs shown that have a value of zero for remnant mass show the corresponding eccentricity and orbital period just before the total disruption. It's worth noting that the only run to have no value for orbital period and an $e > 1$ is the run with a $v_\infty = 1,000$ km/s, comparable to speeds in galactic nuclei.

A possible reason that I think this is occurring is because the approximation for v_{kick} given by equation (2.6.5) shows an inverse proportionality with $M_{rem}^{\frac{1}{2}}$. So higher remnant masses could slow down the kick velocity post tidal disruption to speeds less than necessary to escape the BH. And since we are working with a supergiant star, the remnants produced after tidal

disruption are bound to be more massive than those produced by main sequence stars. Another possible reason is given from Metzger et al. 2022, where they consider energy per unit mass (specific energy) when determining the kick energy of the remnant [34]. Because giants and supergiants have little change in specific energy in comparison to their total orbital energy, it's more difficult to impart a high enough kick velocity to the remnant post-disruption.

5.3.2 Low Mass Loss Case

The first case we will look at in this chapter is one that needs no fabricated justification for evolving in MESA. This simulation was with a $240,000 M_{\odot}$ black hole, giving us a mass ratio of about 10,000, which resulted in a kick energy high enough to extend the orbital period of the remnant to about 1.501×10^7 years. So, while our star is still bound to the black hole post interaction, the orbital period is long enough to observe evolution of the remnant. This particular simulation resulted in a roughly $19 M_{\odot}$ remnant, which is pictured below.

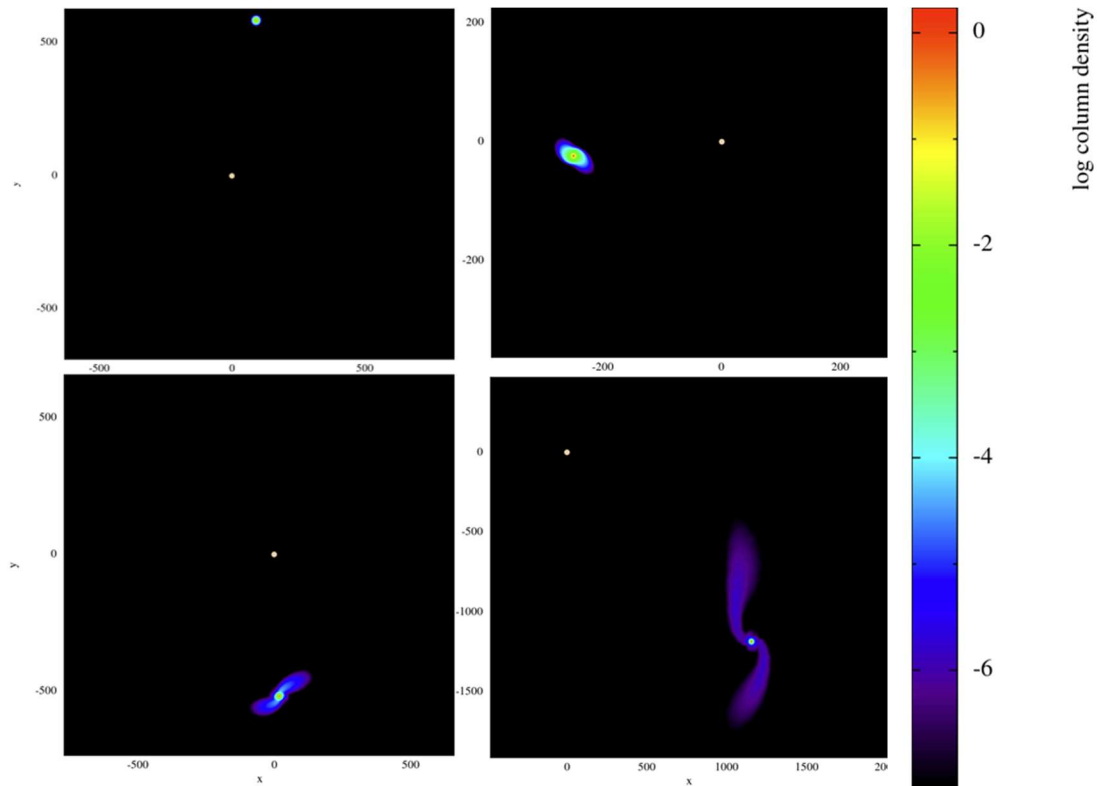


Figure 35: Four frames from the $r_p = r_T$ case with a mass ratio $M_{BH}/M_* = 10,000$. The first frame is the parabolic entry trajectory of the star, second frame is pericenter passage, and the third and fourth frames are of the star leaving the first pericenter passage. The tidal tails formed in tidal disruption are easily seen here. To view a video of this simulation, click this link: <https://www.youtube.com/watch?v=soLA6d8Odo4>

As for how this remnant will evolve in the MESA run, we can take a look at the HR track in the figure below.

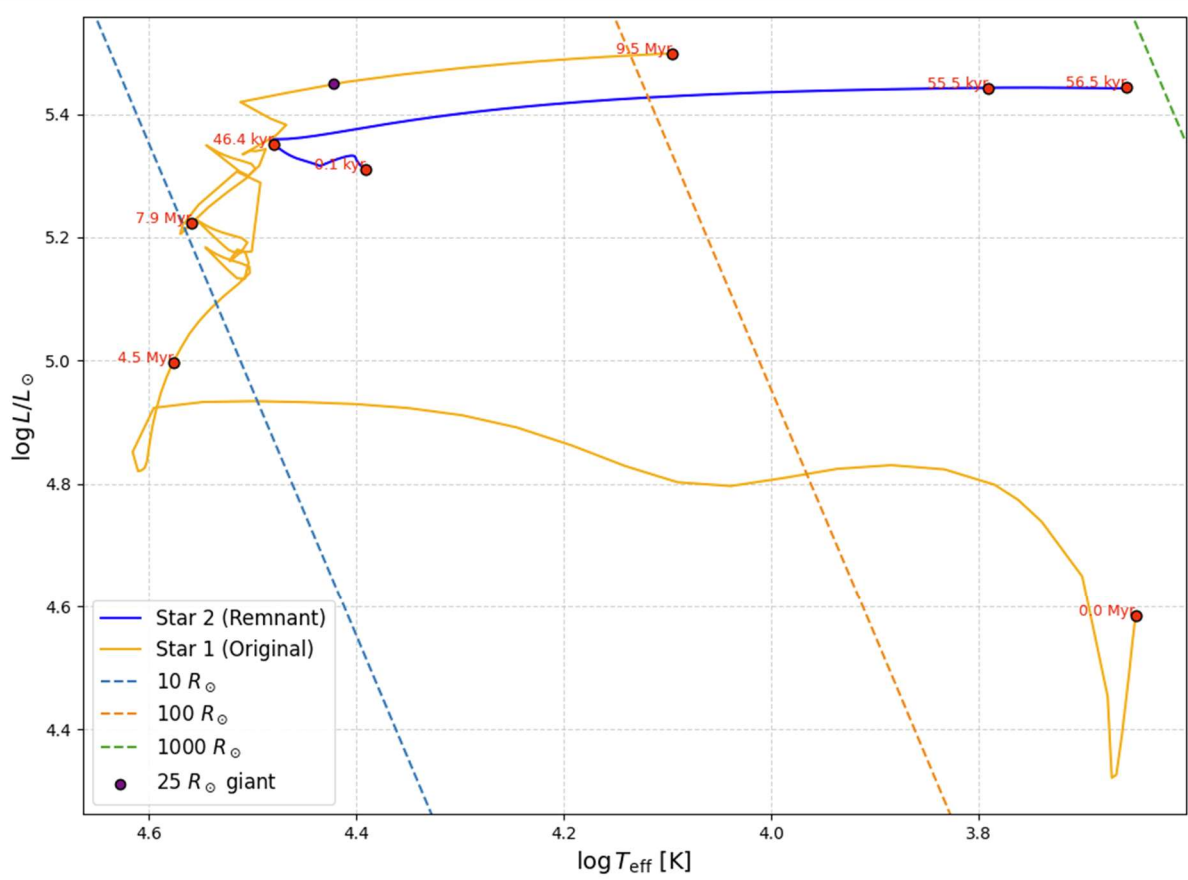


Figure 36: The HR track of our $19 M_{\odot}$ remnant (blue line) compared to the HR track of our original $24 M_{\odot}$ supergiant (orange line). The remnant starts off at around the same size as the supergiant, but a little less luminous and slightly lower in temperature. As the remnant evolves, however, it collapses for a short time, gaining luminosity and temperature until it reaches the giant branch where it starts dramatically increasing in size, denoted by the extreme drop in temperature but with slightly increasing luminosity. The MESA run for our remnant ends at about 56,000 years after the collision took place, with a remnant much larger than our original star.

As for the elemental composition of our remnant compared to a star of the same mass, see the following figure.

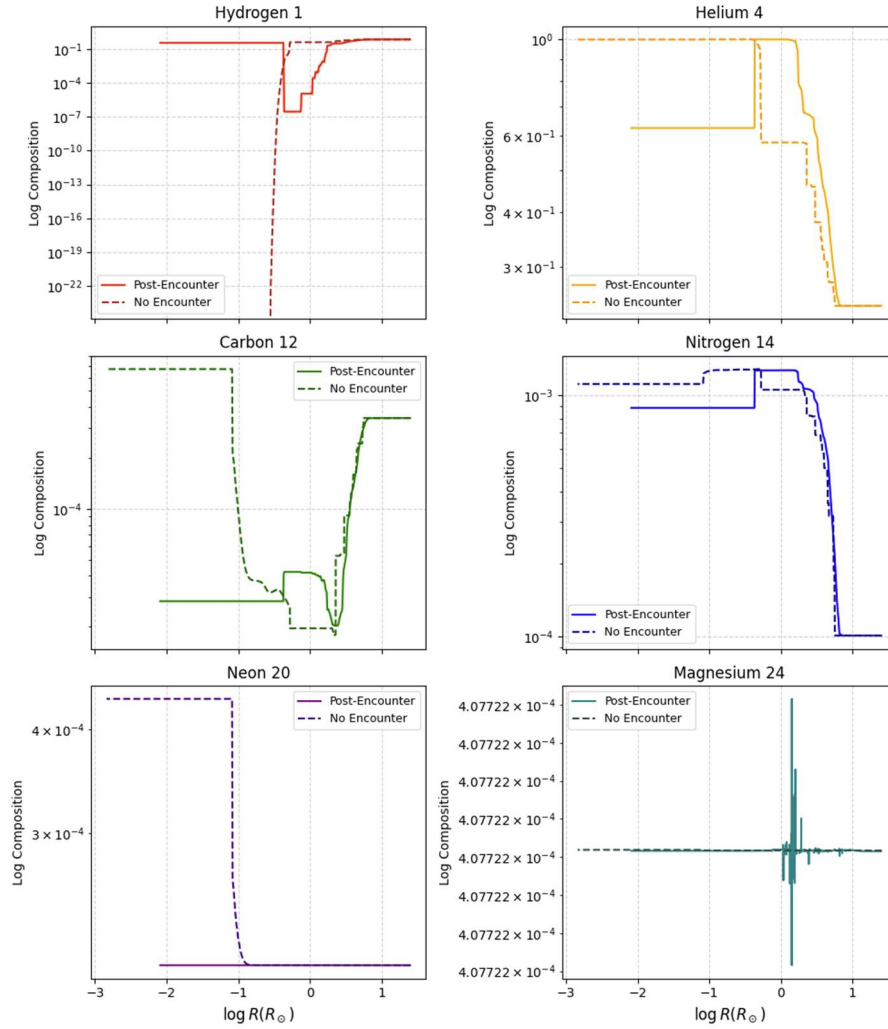


Figure 37: Plots of composition fraction profiles vs radius on a log scale of Hydrogen 1, Helium 4, Carbon 12, Nitrogen 14, Neon 20, and Magnesium 24. Something bizarre here is that the only disparities between element fractions between the two stars are in the inner layers, while the outer layers match up quite well for all elements. Spectroscopic observations of this remnant would look quite like a star of the same mass that is not a collision product.

5.3.3 Higher Mass Loss Case

The final case that we will look at in this paper is the only unbound remnant I was able to create out of all the runs I have done over the course of this project. This was a very close passage between the supergiant star and a black hole of $24,000 M_{\odot}$ with a $r_p/r_T = 0.08$ and a $v_{\infty} = 1,000$ km/s (a speed comparable to the motion of objects in galactic nuclei). As apparent from table 3, it is likely that the only reason that this particular simulation resulted in an unbound

remnant was because the star already had a significant kinetic energy while on its hyperbolic trajectory with the black hole, aiding in imparting energy into kicking the remnant with a high enough velocity to escape the BH. The following figure displays important splash frames from the collision.

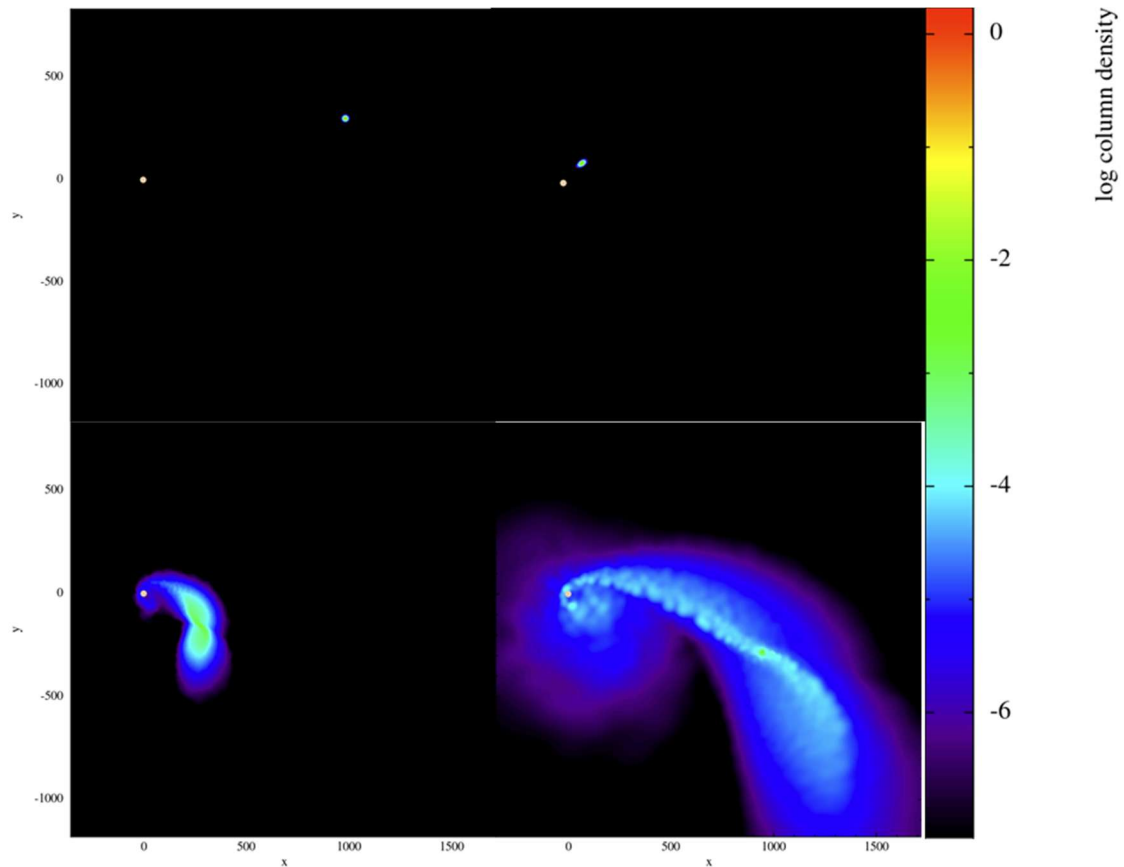


Figure 38: Four frames from the case described above. The first two frames show the star on its way to the black hole on a hyperbolic trajectory as it reaches the first pericenter passage. The last two frames show the tidal disruption of the star, the two tidal tails formed after the interaction, and the unbound remnant on its way out of the system. A video of this collision can be viewed via this link:

<https://www.youtube.com/watch?v=Mq1WRMoDL3w>

Next, we'll take a look at the MESA evolution of this $\sim 10 M_{\odot}$ remnant produced by the above collision. Since this remnant is unbound from the black hole due to the initial velocity aiding the kick, we don't need to worry about an orbital period timescale that will allow evolution of the remnant. The HR track of this remnant along with the original supergiant is pictured in the following figure.

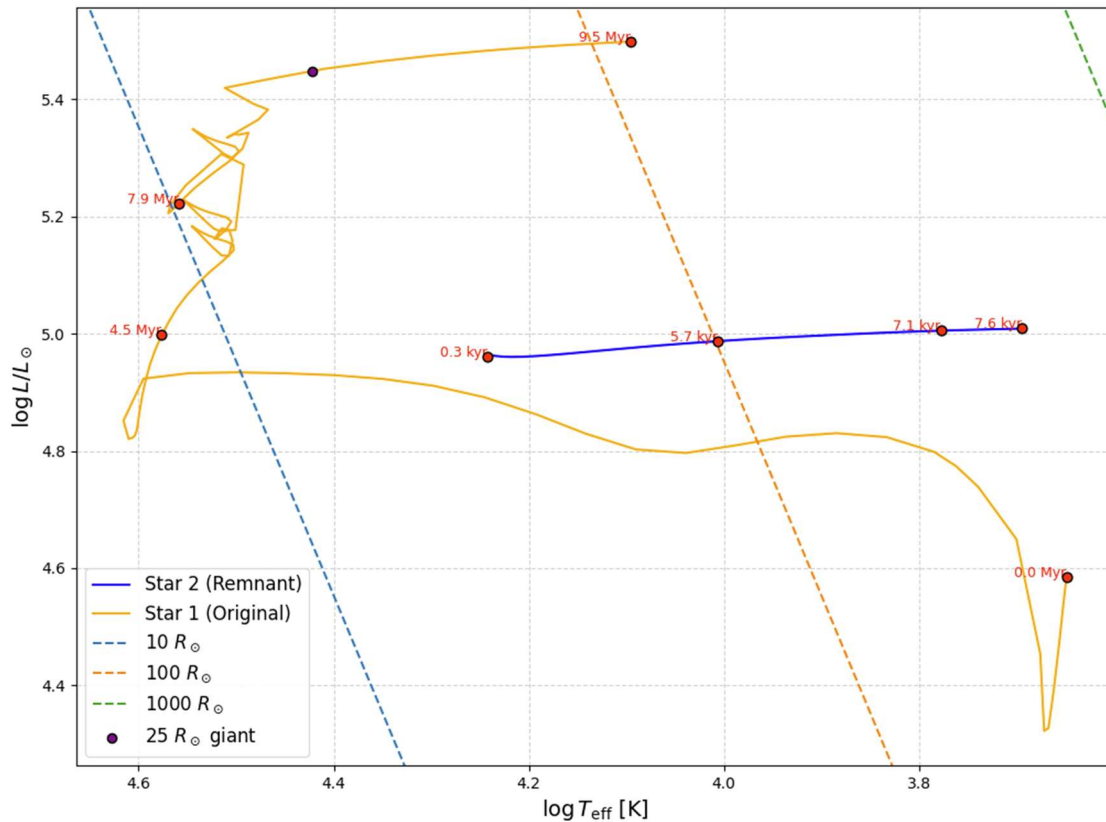


Figure 39: The HR track of the 10 $M_{\odot}$ remnant (blue line) along with our 24 $M_{\odot}$ supergiant (orange line).

The $\sim 58\%$ drop in mass has moved our starting point down quite a bit on the HR diagram. The remnant starts off much lower in luminosity and surface temperature. Over time, the remnant continues to cool off at its surface with a very slight increase in luminosity, yet again signifying that it is expanding. The MESA run for our remnant ends about 7,600 years after the interaction took place.

Next, as per usual, we'll take a look at the composition of our remnant and compare it to a non-TDE product of the same mass to see if the remnant could be easily identified by observations. The composition fraction curves are pictured in the following figure.

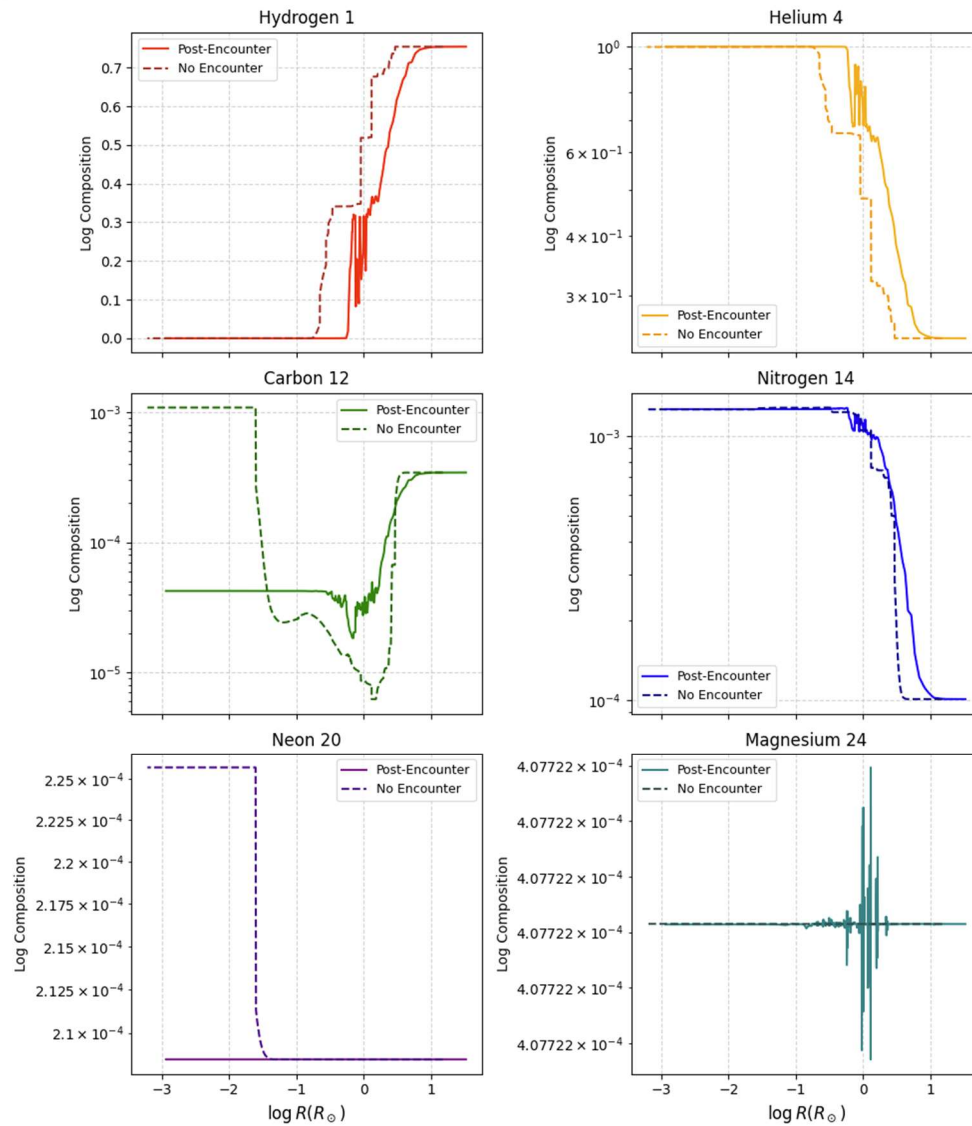


Figure 40: Composition fraction curves on a log scale as a function of log radius for the elements Hydrogen 1, Helium 4, Carbon 12, Nitrogen 14, Neon 20, and Magnesium 24. For ease of viewing, I've undone the log scale for composition fraction of Hydrogen 1, but every other element is represented on a log scale. This was something I was not expecting, while the interiors of the stars show disparity in composition, they line up quite well as we make our way to the surface of the stars. Granted, this remnant is not the core of the star, but rather the core with some of the inner layers still intact so high values for hydrogen and helium are to be expected. However, spectroscopic observations of the remnant might not have any signs of tidal disruption in the star's past.

6 Conclusion and Discussion

Over the course of this project, I've run approximately 60 or more simulations that ended up being of some use to the development of this project, most of which were with the $30 M_{\odot}$ stellar BH. However, the need for adequate justification for evolving collision products in MESA beckoned me to turn my gaze to other options, i.e. raising the mass of the BH. The runs I've taken with the stellar BH were not wasted however, they were integral to analyzing the resulting orbital motion of a star of similar mass and how orbital energy associated with tidal oscillations can lead to tidal capture and inspirals over long periods of time. They were also useful in validating theory posed in other research papers, such as Press & Teukolsky 1977 [26].

As for simulations with higher mass BH's, we found out the glaring differences between partial disruption between main sequence stars and giant stars. Generally, it is much, much more difficult to get a giant stellar remnant to obtain a kick velocity fast enough to send it out of the system on a parabolic trajectory. For starters, the penetration factor for a giant star is much closer to the black hole than for a main sequence star due to the density profile difference. This shrinks the parameter space where an unbound remnant is possible by a large factor. As discussed in section 5.3, other reasons could be due to the mass of the remnant being inversely proportional to v_{kick} or even that the giant's low specific energy cannot add enough to the remnant's kinetic energy post-disruption, as evident from Metzger et al. 2022 [34].

Another result that was surprising, at least to me while conducting these experiments, was the convergence of element composition fractions at the surfaces of PTDE products and normal stars of the same mass and radius. Initially I had expected a disparity between composition fractions at the surfaces of remnant stars, which could then be linked to observations of possible PTDE remnant candidates. However, I could be wrong in this regard and further research could reveal the true answer.

6.1 Future Work

While I think this project has laid down good groundwork, I also think that there are some different directions and improvements a future student could take for this project. First, while I made it out to be that obtaining an unbound remnant from the partial disruption of a giant or supergiant star to be impossible, it could very well happen in a sliver of parameter space that had

happened to slip by me. One way to possibly solve this conundrum is to vary the mass and age of the giant or supergiant star, hence varying the density which will affect the critical penetration factor. This will also widen the range of possibilities from the simulations. Another improvement that I think could be made to this project is investigating the anomalous mixing of elements inside the remnant star's interior as well as the lack of disparity in photosphere element composition fractions for remnant stars vs normal stars. While this could very well reflect the reality of remnant stars, I think looking into the code that produced these results could be useful. Another way to verify this is to create remnant stars with so much mass stripped off that the remnant is basically just the naked core of the star, which will undoubtedly have an interesting photosphere composition.

Appendix A: Important Units in *StarSmasher*

In the SPH code *StarSmasher*, the units of certain parameters are expressed out of three fundamental units for mass, length, and the gravitational constant from Newtonian gravity, covered in section 2.1.

Mass: $M_{\odot} = 1.989 \times 10^{30}$ kg (one solar mass)

Length: $R_{\odot} = 6.597 \times 10^8$ m (one solar radius)

Gravitational Constant: $G = 6.6743 \times 10^{-11}$ Nm²/kg²

The value of each of these units is 1 in the code and they can be rearranged mathematically to obtain units for other parameters. Some of the useful units are listed below.

Density: $\rho = \frac{M_{\odot}}{R_{\odot}^3} = 6927.7998 \frac{\text{kg}}{\text{m}^3}$

Energy: $E = \frac{GM_{\odot}^2}{R_{\odot}} = 4.0025 \times 10^{41}$ J

Velocity: $v = \sqrt{\frac{GM_{\odot}}{R_{\odot}}} = 437$ km/s

Time: $t = \sqrt{\frac{R_{\odot}^3}{GM_{\odot}}} = 26.54$ minutes

Appendix B: A Guide to Evolving Remnants and Other Collision Products

A large portion of this research is indebted to Charlie Gibson ²⁵, who developed the post-processing code *sph-to-mesa* that allows the user to take data from an SPH collision and import it over to MESA where the evolution of potential collision products can be run. Without Charlie's talents and knowledge, arguably the most interesting part of this research project would not be possible. The following guide for running the post-processing code could be useful to anyone who intends to evolve collision products in MESA. For more information on *sph-to-mesa* and its development, I highly recommend you read Charlie Gibson's comp.

First, we're going to copy the *sph-to-mesa* directory over to your home directory on gonzales.

```
cp -pr ~gibson01/sph-to-mesa .
```

Next, we'll want to copy *splot.py* to your bin directory

```
cp -p ~gibson01/sph-to-mesa/splot.py bin/splot
```

```
chmod a+x bin/splot
```

The next thing we need to do is evaluate your stellar relaxation in the directory where you relaxed your star model.

```
splot
```

```
11
```

```
last_output_#
```

```
last_output_#
```

```
1
```

DT (If the eosDT causes problems down the line, consider changing it to DE (density-specific internal energy))

1

This will create an sph.composition file. Next, copy sph.composition over to the directory where you collided this star model.

```
cp -p /path/to/relaxation/sph.composition /path/to/collision
```

Then make a backup of sph.composition

```
cp -p sph.composition sph.composition_orig
```

From here, open sph.composition and add a line at the end with eight zeros, separated by a space and hitting enter at the end. This will create an index that represents the black hole in a TDE or PTDE simulation.

This next step can be rather tedious, if you have an reat parameter in you sph.input file, then we're going to have to remove lines in sph.composition that correspond to the "eaten" particles.

```
grep throwing log*.sph
```

This will return several indices for eaten particles. Remove them in sph.composition by typing:

```
emacs +particle_index_# sph.composition
```

to enter the file at that index, then type ctrl k, backspace, then ctrl x – ctrl s to save the file then ctrl x – ctrl c to exit the file. Do this for every eaten particle index!

Next, we are going to run splot again to determine the boundness of each particle

```
splot
```

```
4
```

```
0
```

```
last_output_#
```

```
1
```

(Here you can type whatever you want, it could be 1, 2, 100, 1000. It depends on how many comp files you want and how much disk space is left on gonzaless)

0

It's worth noting that the BH in a simulation can be incorrectly assigned as a bound particle to the star. To fix this, go into the last comp.sph file produced after this step and type ctrl down arrow to go to the bottom of the file, then replace the last "1" at the end of the line with a "4". It's also worth noting that we set the fraction of internal energy to zero when determining bound particles because then the code only considers particles with gravitational and potential energy while not considering particles with thermal energy.

Next, we'll run splot again to analyze the structure of the collision

```
splot
3
last_output_#
last_output_#
1
DT (DE if DT doesn't work)
1
0
```

Next, we can finally generate the MESA input files that will get copied over to a machine with MESA downloaded.

```
splot
10
last_output_#
last_output_#
1
DT (DE if DT doesn't work)
```

1

It's finally time to run the star in MESA! First, we'll need to make a directory for our MESA run on another machine, probably carrv122, and copy the directory raw_mesa_template from Charlie Gibson's data directory.

```
cp -pr /data/gibson01/raw_mesa_template/ your_mesa_directory
```

Then we can copy all of the files generated in MESA_inlist_files to your mesa directory

```
cd MESA_inlist_files
```

```
scp -p * carrv122:/path/to/your_mesa_directory/raw_mesa_template
```

Finally, we can begin the MESA run in our new mesa directory!

```
./clean
```

```
./mk
```

```
./rn
```

After your run is finished, if you want to see the HR track of your remnant compared to your original star, feel free to run the python code in my carrv122 directory

```
cp -p /data/johnson03/both_hr_use.py /your/mesa/dir
```

```
python3 both_hr_use.py
```

Just make sure to change the file paths in the code to match that of your remnant and original star MESA runs.

Another python code you can use is one that will graph the element compositions as a function of radius throughout your collision product and compare it to that of a star of the same mass.

This code can be copied from:

```
cp -p /data/johnson03/TDE_composition.py /your/mesa/dir/LOGS
```

```
python3 TDE_composition.py
```

Or if you just want to plot the composition fractions of different elements in just your TDE remnant, you can run the following code:

```
cp -p /data/johnson03/one_star_composition.py /you/mesa/dir/LOGS
```

```
python3 one_star_composition.py
```

Once again, just make sure that the file paths in the code match up with the file paths to your remnant MESA run and your equivalent mass star MESA run.

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